Name: _____

Date: _____

Collaborators: _____

Question:	1	2	3	Total
Points:	25	25	25	75
Score:				

Instructor/grader comments:

The method of residues

1. (25 points) Calculate the following integral for real a, $|a| < 1$, and integer n, $n \geq 0$:

$$I_n(a) = \int_{-\pi}^{\pi} \frac{\cos(n\varphi)}{1 - 2a\cos(\varphi) + a^2}\, \mathrm{d}\varphi.$$

   Sketch the integration contour. Indicate the position(s) of the pole(s) of the integrand. To verify your answer, consider the limit $a \to 0$, $n \neq 0$. (Note that $I_n(0) = 0$ for $n \neq 0$.)

   Hint: Consider the integral

$$J_n(a) = \int_{-\pi}^{\pi} \frac{e^{in\varphi}}{1 - 2a\cos(\varphi) + a^2}\, \mathrm{d}\varphi.$$

2. (25 points) Calculate the integral for $a > 0$:

$$I(a) = \int_0^{\infty} \frac{\mathrm{d}x}{a + x^3}.$$

   Use the integration contour shown in Fig. 1 where $R \to \infty$. Sketch the position(s) of the pole(s) of the integrand.

   To verify your solution, use a computer algebra system to plot a graph of $I(a)$ for $1 \leq a \leq 8$. The expected graph is shown in Fig. 2.

   Attach a printout of your CAS session.

3. (25 points) Calculate the integral for $a > 0$:

$$I(a) = \int_0^{\infty} \frac{\mathrm{d}x}{(a^2 + x^2)^2}.$$

   Sketch the integration contour. Indicate the position(s) of the pole(s) of the integrand.

   To verify your solution, use a computer algebra system to plot a graph of $I(a)$ for $3 \leq a \leq 8$. The expected graph is shown in Fig. 3.

   Attach a printout of your CAS session.

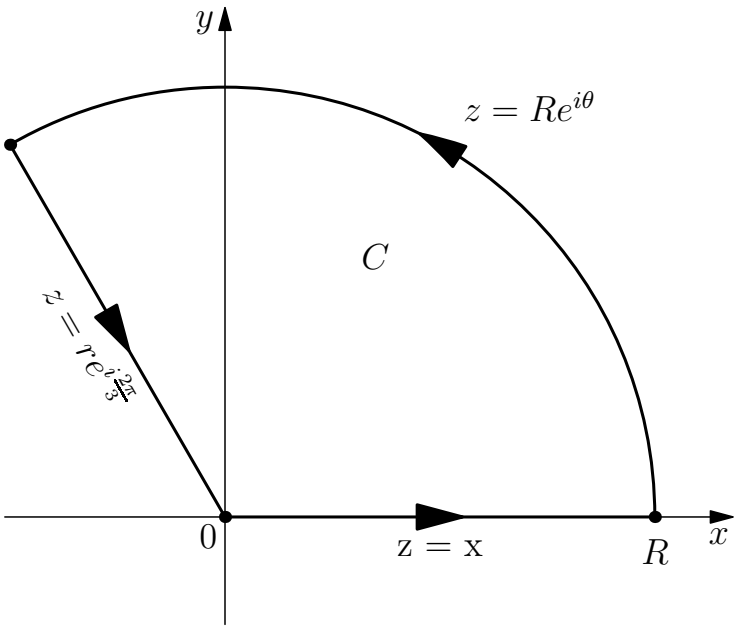

Figure 1: Integration contour for Problem 2.

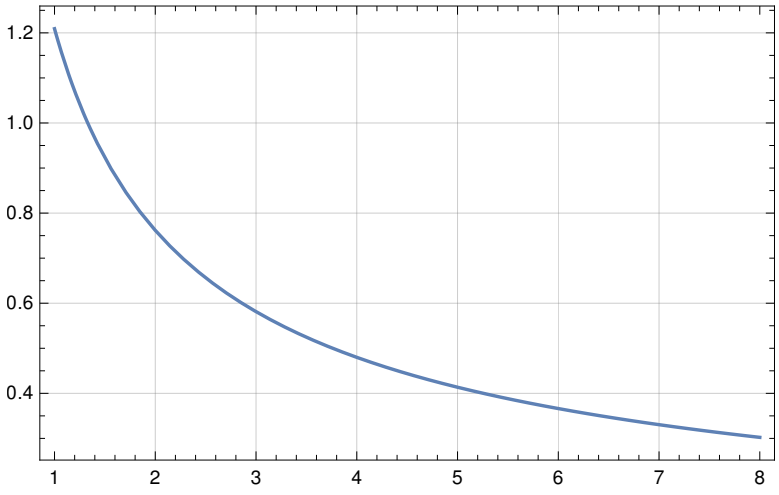

Figure 2: Expected graph in Problem 2

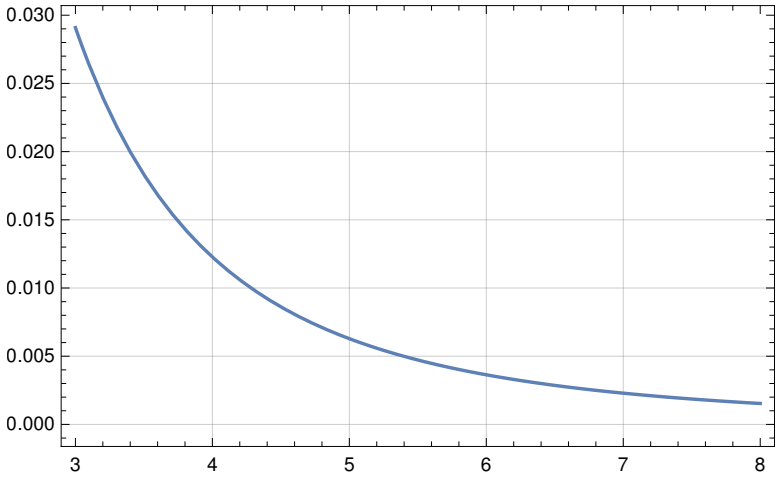

Figure 3: Expected graph in Problem 2