Name: ______________________________

Date: ______________________________

Collaborators: ____

Question:	1	2	3	Total
Points:	10	25	25	60
Score:				

Instructor/grader comments:

Cauchy-Riemann equations

1. (10 points) Verify that that the following function

$$u(x,y) = x^2 - 2y^2$$

cannot be a real part of any complex function $f(z)$.

2. (25 points) Use Cauchy-Riemann equations to find the analytic function $f(z)$, $z = x+iy$, such that its real part is as following:

$$\operatorname{Re} f(z) = u(x,y) = e^x \sin y,$$

and

$$f(i\pi) = 0.$$

Express the result for $f(z)$ as a **function of z only.**

Answer: $f(z) = -i\,(e^z + 1)$.

The Cauchy integral theorem

3. (25 points) Evaluate the integral in terms of Gamma function:

$$I = \int_0^\infty \sin\left(x^3\right) \mathrm{d}x$$

Answer: $I = \frac{1}{2}\Gamma\left(\frac{4}{3}\right)$.

Hints: consider the integral

$$\oint_C e^{-z^3}\,\mathrm{d}z$$

along the contour C sketched in Fig. 1; use the Euler formula; use the relation

$$\int_0^\infty e^{-x^3}\,d\mathrm{x} \equiv \Gamma\left(\frac{4}{3}\right),$$

where Γ is gamma function.

Accept without a proof that the integral over the circular arc in Fig. 1 goes to 0 as $R \to \infty$.

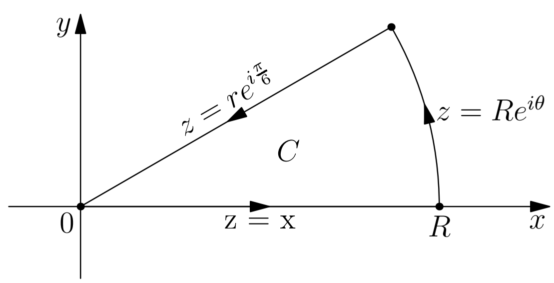

Figure 1: Integration contour for Problem 3. $(R \to \infty)$.