Name: _____

Date: _____

Collaborators: _____

Question:	1	2	3	4	5	6	Total
Points:	15	15	10	10	10	10	70
Score:							

Instructor/grader comments:

Euler's formula

1. (15 points) Evaluate the following sum:

$$C(\theta) = \cos(\theta) + \cos(2\theta) + \ldots + \cos(n\theta) = \sum_{k=1}^{n} \cos(k\theta). \qquad (A)$$

Simplify your answer. Your final expression must contain only trigonometric functions and possibly constants. Make sure that in the limit $\theta \to 0$ you get $C(0) = n$.

1. To verify your result, plot on the same graph, for $n = 5$ and for $-\pi \le \theta \le \pi$, the direct sum (A) and your answer. The resulting graph should look similar to Figure 1.
2. Print your Mathematica session (use "File" → "Save as" menu) and attach the printout to the rest of your homework.

Hints:

1. Recall that
$$\cos\theta = \mathrm{Re}\left(e^{i\theta}\right).$$
2. The sum of real parts equals to the real part of the sum.
3. Recall the expression for the sum of the geometric series:
$$a + a^2 + a^3 + \ldots + a^n = \sum_{k=1}^{n} a^k = \frac{a(1-a^n)}{1-a}.$$

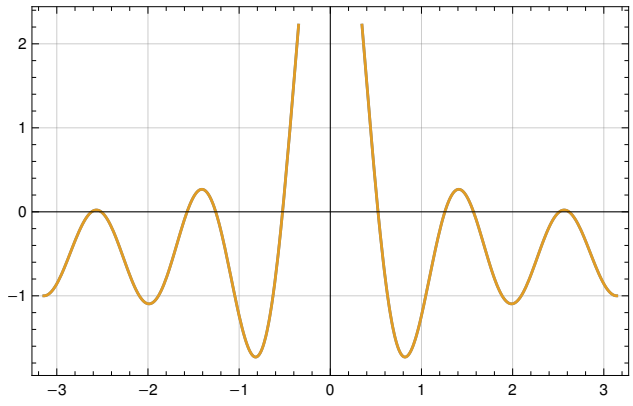

Figure 1: Expected graph in Problem 1

The problems 2–6 below are not calculator, CAS, or computer problems. No credits will be given if those tools are used for solutions.

Gamma function

2. (15 points) Evaluate the integral in terms of Gamma function. Simplify the expression as much as possible.

$$I = \int_0^\infty e^{-x^4} \mathrm{d}x$$

Answer: $I = \Gamma\left(\frac{5}{4}\right)$

Complex numbers

3. (10 points) Find the coordinate and the polar form of the following complex number:

$$Z = \left(\frac{\sqrt{2} - i\sqrt{2}}{1 - i\sqrt{3}}\right)^{26}.$$

Answer: $Z = e^{i\frac{\pi}{6}} = \frac{\sqrt{3}}{2} + \frac{i}{2}$

4. (10 points) Find the absolute value of the following complex number:

$$Z = \left(\frac{3}{5} + i\frac{4}{5}\right)^n \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^m$$

for arbitrary positive integers n and m.

Answer: $|Z| = 1$.

5. (10 points) Find the values of $Z = \left(\sqrt{i}\right)^i$.

Answer: $Z = e^{-\frac{\pi}{4} - \pi n}$, where $n = 0, \pm 1, \pm 2, \ldots$

6. (10 points) Find the coordinate and the polar forms of the solutions of the equation:

$$z^4 = \sqrt{3} - i.$$

How many roots are there?

Answer: the solutions of the equations are sketched in Fig. 2.

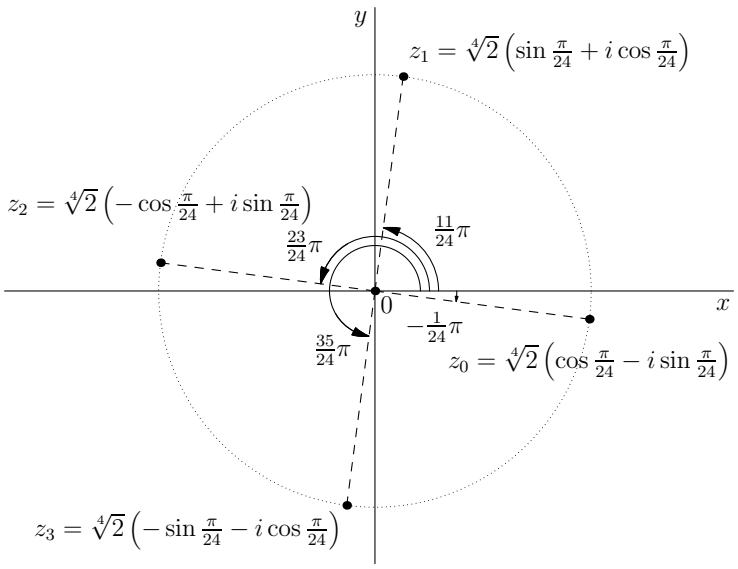

Figure 2: Roots of the equation $z^4 = \sqrt{3} - i$.