Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	15	15	15	25	70
Score:					

Instructor/grader comments:

Computer algebra

1. (15 points) Consider the following integral:

$$I(x) = \int_{\frac{1}{2}}^{\pi-\frac{1}{2}} e^{-(y-\pi/2)^6} \sin^x(y)\,\mathrm{d}y.$$

   a. Verify that Mathematica cannot obtain an analytic expression for the integral. Use Mathematica function `Integrate[]`.

   b. Define the function, `f[x_]`, that evaluate the integral numerically. Use Mathematica function `NIntegrate[]`.

   c. Plot on the same graph, for $10 \le x \le 50$, your function and the following approximation to the integral:

$$g(x) = \sqrt{\frac{2\pi}{x}}.$$

   (We are going to learn how to obtain approximations for this and similar integrals later in the course.)

   The resulting graph should look similar to Figure 1.

   d. Print your Mathematica session (use "File" → "Save as" menu) and attach the printout to the rest of your homework.

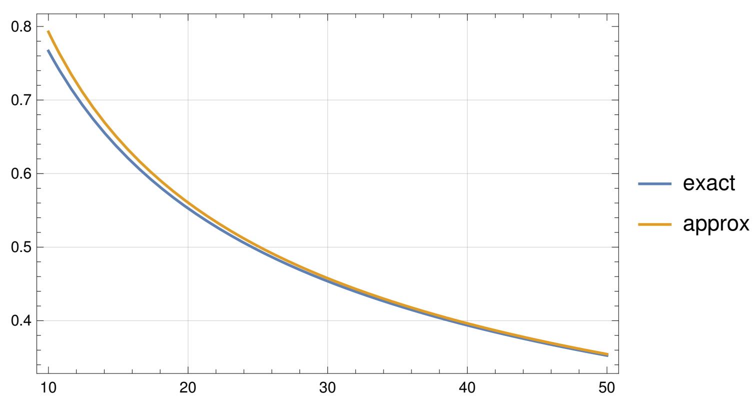

Figure 1: Expected graph in Problem 1

2. (15 points) Consider the following two-point boundary value problem:

$$\epsilon y'' + 2y' + e^y = 0, \quad y(0) = 0, \quad y(1) = 0, \quad \epsilon = 1/20.$$

   a. Verify that Mathematica cannot obtain an analytic expression for the integral. Use Mathematica function `DSolve[]`.

   b. Solve the boundary value problem numerically. Use Mathematica function `NDSolve[]`.

   c. Plot on the same graph, for $0 \le x \le 1$, the numerical solution of the boundary value problem and the following approximation to the solution:

$$g(x) = \log\left(\frac{2}{1+x}\right) - \log(2)\exp\left(-\frac{2x}{\epsilon}\right)$$

   (We are going to learn how to obtain approximations for solutions of differential equations later in the course.)

   The resulting graph should look similar to Figure 2.

   d. Print your Mathematica session (use "File" → "Save as" menu) and attach the printout to the rest of your homework.

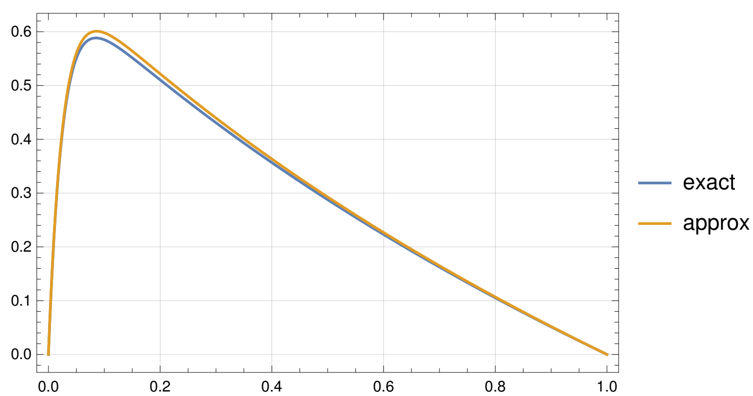

Figure 2: Expected graph in Problem 2

3. (15 points) Consider the following expression:

$$\ln(n!),$$

where $n! \equiv 1 \times 2 \times 3 \cdots (n-1) \times n$ is the factorial of integer n.

a. Verify that Mathematica cannot simplify the expression to obtain a formula usable for (astronomically) large n.

b. Define the function, `f[x_]`, that evaluate the above expression. Use Mathematica function `Factorial[]`.

c. Plot on the same graph, for $1 \leq x \leq 5$, your function and the following approximation:

$$g(x) = \log(2\pi)/2 + (1/2 + x) * \log(x) - x$$

(We are going to learn how to obtain approximations for sums and products later in the course.)

d. The resulting graph should look similar to Figure 3.

e. Print your Mathematica session (use "File" → "Save as" menu) and attach the printout to the rest of your homework.

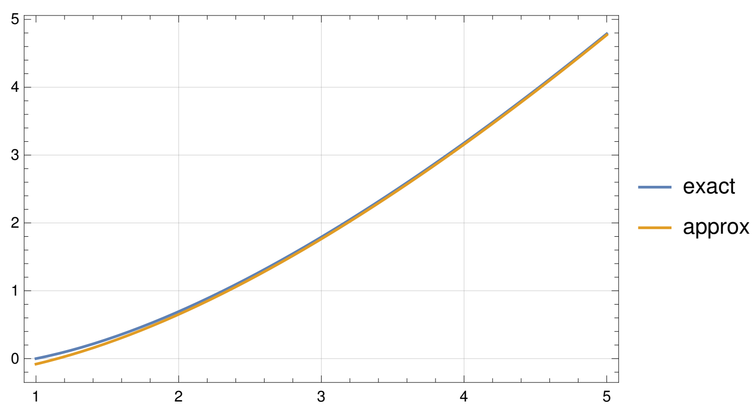

Figure 3: Expected graph in Problem 3

Course concepts

4. (a) (15 points)

   ☐ I've watched in full the video recording of R. Feynman's lecture *The relation of Mathematics and Physics*.

   (b) (10 points)

   ☐ I've read the Introduction, pp. 9–13, to the lecture notes *Physical Mathematics*, by Michael P. Brenner.

Sign and date here: ______________________________