Name: _____

Date: _____

Collaborators: _____

Question:	1	2	3	4	Total
Points:	25	15	15	30	85
Score:					

Instructor/grader comments:

Laplace method for integrals

1. (a) (20 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^\pi \frac{\mathrm{d}x}{(x^2+1)^\lambda}.$$

Clearly describe the location of the maximum of the integrand, and the approximation you used for the integrand in the vicinity of the maximum.

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $10 < \lambda < 100$. Attach a printout of your CAS session.

The expected graph is shown in Fig. 1.

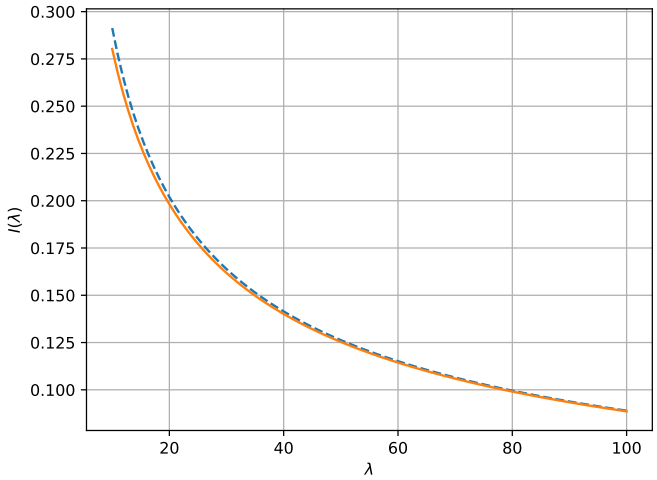

Figure 1: Expected result in Problem 1.

2. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_{-3}^{2} e^{-\lambda \sinh^2 x} \, \mathrm{d}x.$$

Clearly describe the location of the maximum of the integrand, and the approximation you used for the integrand in the vicinity of the maximum.

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $5 < \lambda < 100$. Attach a printout of your CAS session.

The expected graph is shown in Fig. 2.

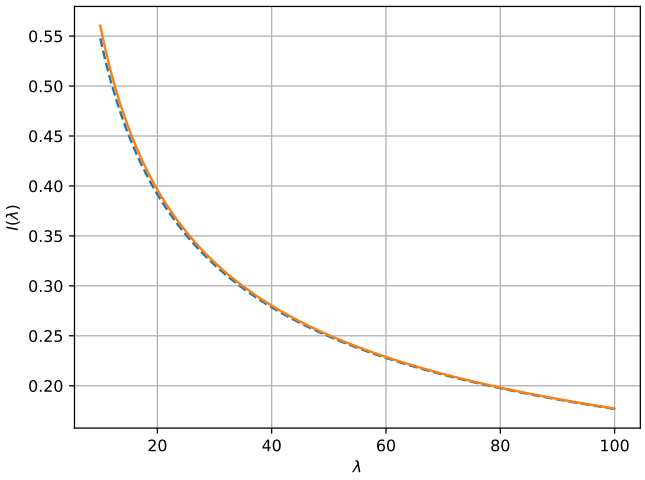

Figure 2: Expected result in Problem 2.

3. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^3 \frac{e^{-\lambda x}}{\cosh x} \mathrm{d}x.$$

Clearly describe the location of the maximum of the integrand, and the approximation you used for the integrand in the vicinity of the maximum.

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $5 < \lambda < 25$. Attach a printout of your CAS session.

The expected graph is shown in Fig. 3.

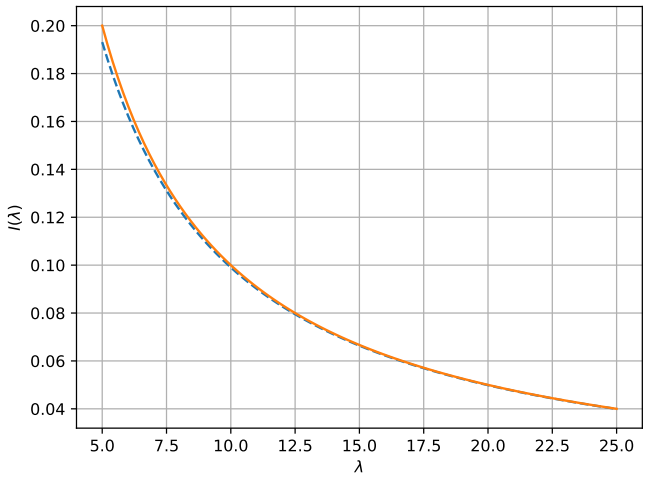

Figure 3: Expected result in Problem 3.

4. (a) (25 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^\infty e^{-\lambda x - \frac{4}{x^2}} \mathrm{d}x.$$

Hint: the integrand in this problem has a *moving maximum*. You need to change the integration variable so that the maximum of the integrand occurs at a fixed point independent on λ.

(b) (5 points) Use a Computer Algebra System to plot on the same graph the numerical value of the integral and your approximation for $3 < \lambda < 7$. Use log scale for y axis. Attach a printout of your CAS session.

The expected graph is shown in Fig. 4.

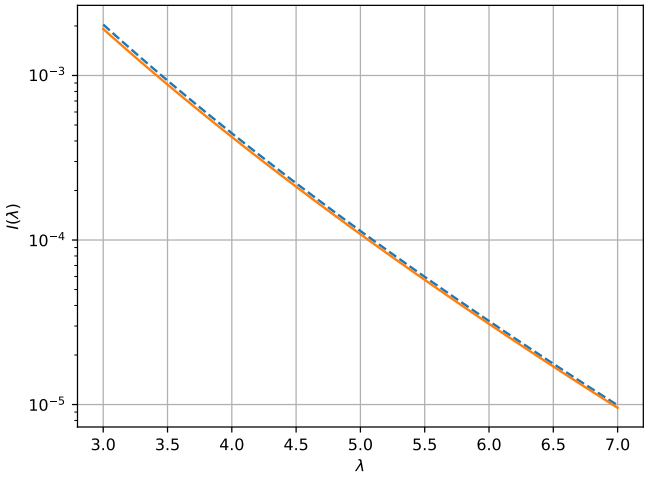

Figure 4: Expected result in Problem 4.