Name: _____

Date: _____

Collaborators: _____

Question:	1	2	3	Total
Points:	20	25	25	70
Score:				

Instructor/grader comments:

Cauchy-Riemann equations

1. (20 points) Use Cauchy-Riemann equations to find the analytic function $f(z)$, $z = x+iy$, such that its real part is as following:

$$\mathrm{Re}\, f(z) = u(x,y) = e^x \sin y,$$

and

$$f(i\pi) = 0.$$

Express the result for $f(z)$ as a **function of z only.**

Answer: $f(z) = -i\,(e^z + 1)$.

The Cauchy integral theorem

2. (25 points) Evaluate the integral

$$I = \int_0^\infty \sin\left(x^3\right) \mathrm{d}x$$

Hints: consider the integral

$$\oint_C e^{-z^3}\, \mathrm{d}z$$

along the contour C sketched in Fig. 1; use the Euler formula; use the fact that

$$\int_0^\infty e^{-x^3}\, d\mathrm{x} \equiv \Gamma\left(\frac{4}{3}\right),$$

where Γ is gamma function.

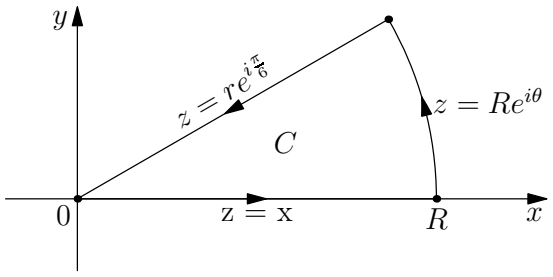

Figure 1: Integration contour for Problem 2. ($R \to \infty$) .

Answer: $I = \frac{1}{2}\Gamma\left(\frac{4}{3}\right)$.

Integral to stump a computer algebra system

3. (a) (20 points) Construct an definite integral that you can evaluate analytically but a computer algebra can not. Use the method described in the handout "The integral that stumped Feynman". Do not use the integrands similar to ones discussed in the handout. Use a computer algebra system for finding the real and the imaginary parts of your complex expressions.

   (b) (5 points) To verify your result, numerically evaluate your integral and your answer. Use a computer algebra system for numerics.

   Enclose a printout of you computer algebra session.