Name: _____

Date: _____

Collaborators: _____

Question:	1	2	3	4	5	6	Total
Points:	20	15	15	15	15	20	100
Score:							

Instructor/grader comments:

Method of stationary phase

1. (a) (15 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^1 \cos(\lambda x^p)\, \mathrm{d}x$$

for p real and $p > 1$.

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation for $p = 3$ and $10 < \lambda < 100$.

The expected graph is shown in Fig. 1.

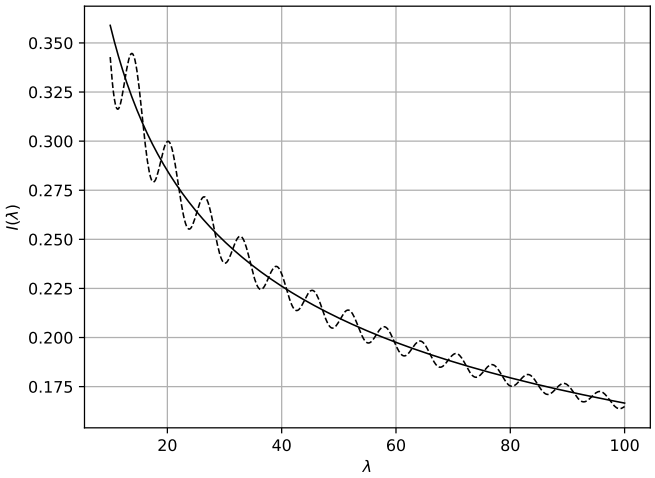

Figure 1: Expected result in Problem 1 (solid line – asymptotics, dashed line – numerically evaluated integral).

2. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^\infty \cos\left(\lambda x^2 - x\right) \mathrm{d}x.$$

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation for $16 < \lambda < 1024$.

The expected graph is shown in Fig. 2.

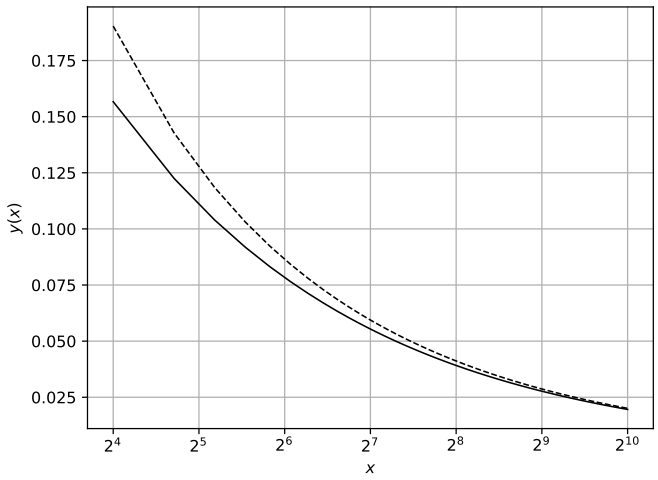

Figure 2: Expected result in Problem 2 (solid line – asymptotics, dashed line – numerically evaluated integral).

3. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^\pi \cos\left(\lambda \cos(x)\right) \mathrm{d}x.$$

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation for $1 < \lambda < 30$.

The expected graph is shown in Fig. 3.

Integration by parts

4. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_1^\infty \cos\left(\lambda x^2\right) \mathrm{d}x.$$

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation for $10 < \lambda < 100$.

The expected graph is shown in Fig. 4.

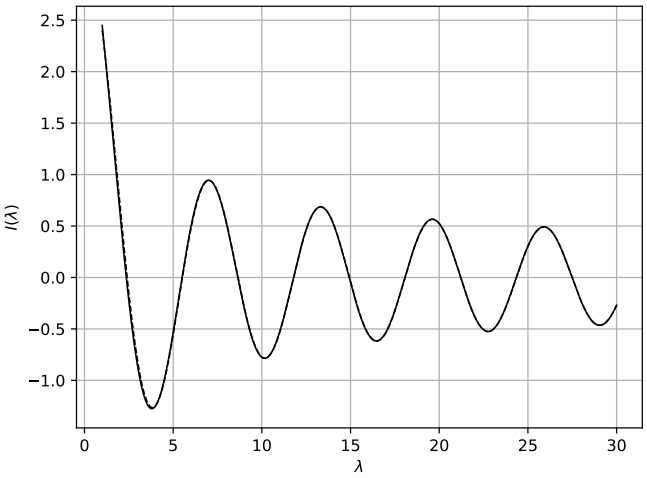

Figure 3: Expected result in Problem 3 (solid line – asymptotics, dashed line – numerically evaluated integral).

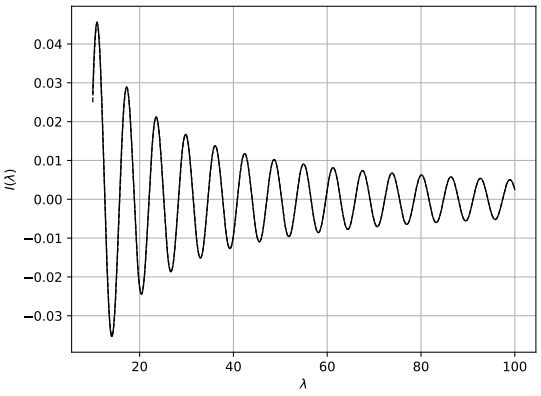

Figure 4: Expected result in Problem 4 (solid line – asymptotics, dashed line – numerically evaluated integral).

5. (a) (10 points) Improve the approximation for the integral you considered in Problem 1 by evaluating the first correction term for $\lambda \to \infty$:

$$I(\lambda) = \int_0^1 \cos\left(\lambda x^3\right) \mathrm{d}x.$$

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation for $10 < \lambda < 100$. The expected graph is shown in Fig. 5.

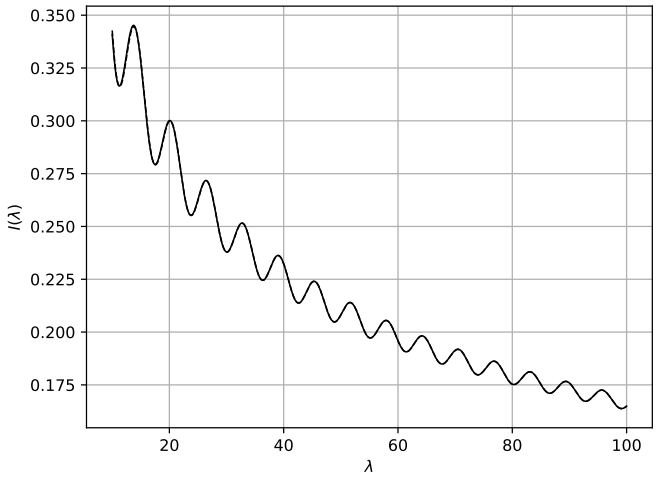

Figure 5: Expected result in Problem 5 (solid line – asymptotics, dashed line – numerically evaluated integral).

6. (a) (15 points) Use the method of stationary phase to find the solution of the differential equation from Problem 1 HW6 for $t \gg 1$.

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation for $1 < t < 150$.

The expected graph is shown in Fig. 6.

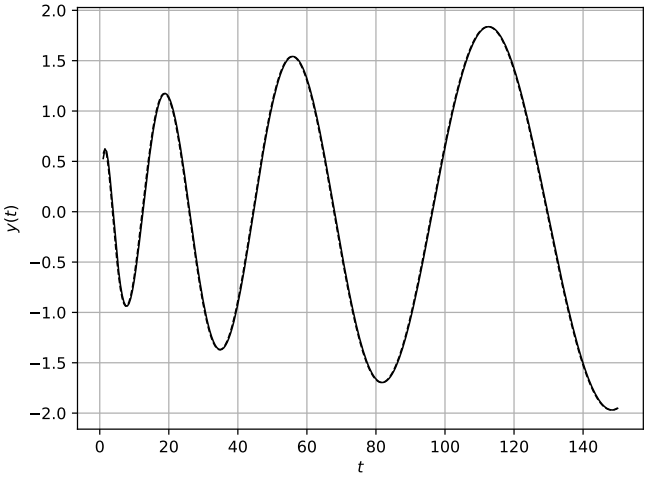

Figure 6: Expected result in Problem 6 (solid line – asymptotics, dashed line – numerically evaluated integral).