Name: _____

Date: _____

Collaborators: _____

Question:	1	2	3	4	5	6	7	Total
Points:	15	20	20	15	15	15	25	125
Score:								

Instructor/grader comments:

Laplace method for ODEs

1. (15 points) Write a contour integral solution of the following equation

$$t\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+y=0$$

As an integration contour chose the unit circle around the origin

Laplace method for integrals

2. (a) (15 points) Find the leading term of the asymptotics of the following integral for $\lambda\to\infty$:

$$I(\lambda)=\int_0^\pi \frac{\mathrm{d}x}{(x^2+1)^\lambda}.$$

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation for $10<\lambda<100$.

The expected graph is shown in Fig. 1.

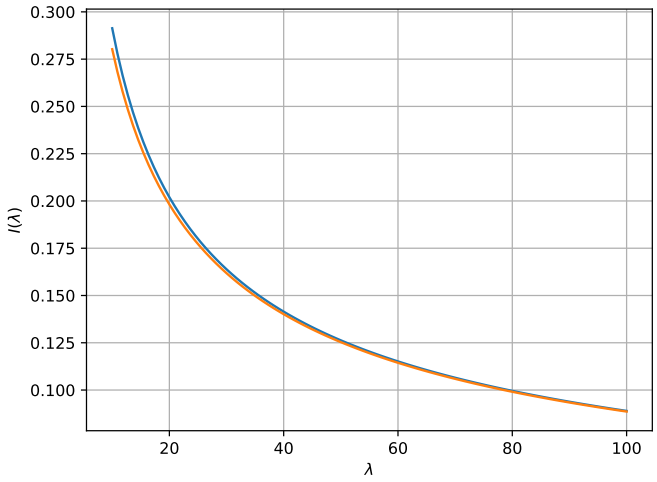

Figure 1: Expected result in Problem 2.

3. (a) (15 points) Find the leading term of the asymptotics (for $x\to\infty$) of the *complementary error function* defined by the integral :

$$\mathrm{erfc}(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,\mathrm{d}t.$$

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation for $1 < x < 4$. Use log scale for y axis.

The expected graph is shown in Fig. 2.

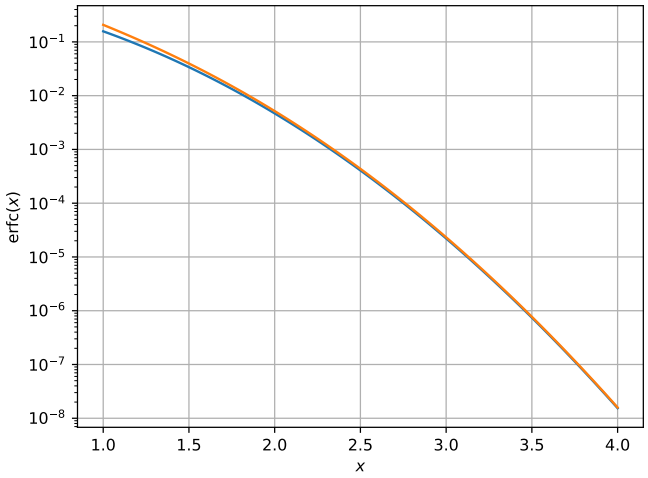

Figure 2: Expected result in Problem 3.

4. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_{-3}^{2} e^{-\lambda \sinh^2 x}\, \mathrm{d}x.$$

On the same graph plot the numerical value of the integral and your approximation vs. λ for $5 < \lambda < 25$.

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation for $1 < x < 4$.

The expected graph is shown in Fig. 3.

5. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_{0}^{3} \frac{e^{-\lambda x}}{\cosh x} \mathrm{d}x.$$

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation vs. λ for $5 < \lambda < 25$.

The expected graph is shown in Fig. 4.

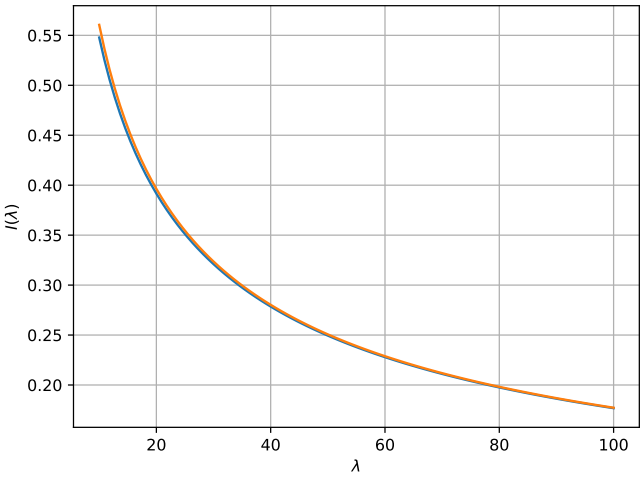

Figure 3: Expected result in Problem 4.

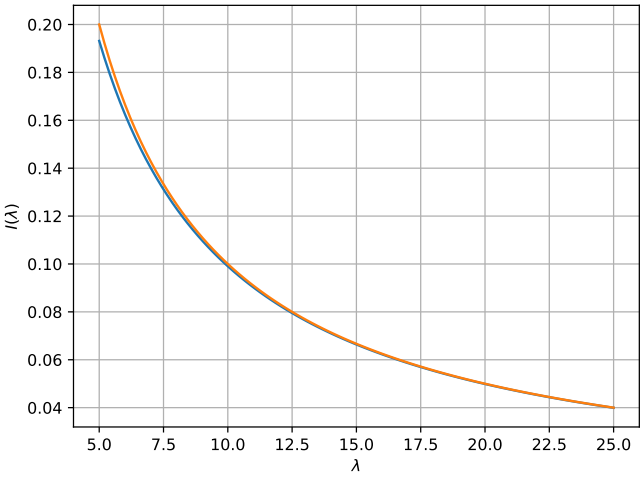

Figure 4: Expected result in Problem 5.

6. (a) (10 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^1 e^{-\lambda \tan x} \mathrm{d}x.$$

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation vs. λ for $10 < \lambda < 25$.

The expected graph is shown in Fig. 5.

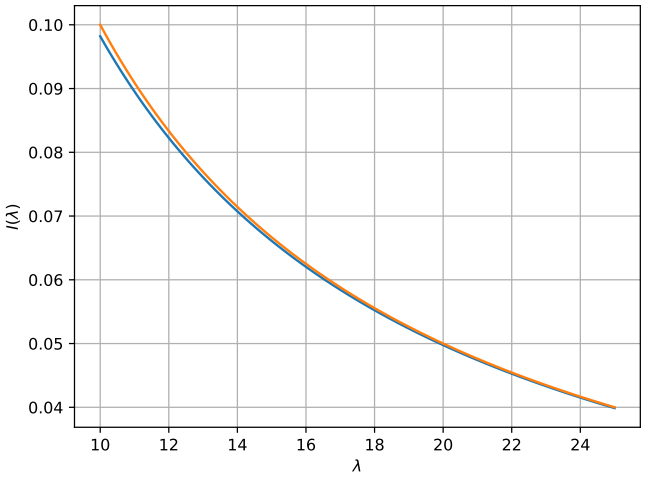

Figure 5: Expected result in Problem 6.

7. (a) (20 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^\infty e^{-\lambda x - \frac{4}{x^2}} \mathrm{d}x.$$

Hint: the integrand in this problem has a *moving maximum*.

(b) (5 points) On the same graph plot the numerical value of the integral and your approximation vs. λ for $3 < \lambda < 7$. Use log scale for y axis.

The expected graph is shown in Fig. 6.

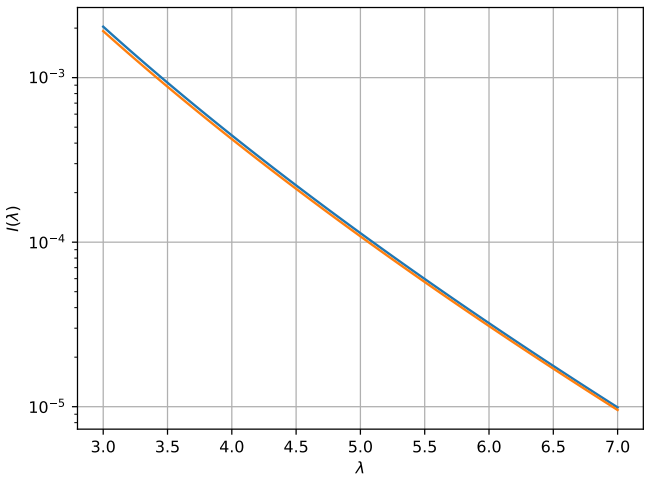

Figure 6: Expected result in Problem 7.