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Date: _____

Question:	1	2	Total
Points:	45	55	100
Score:			

Instructor comments:

1. (45 points) Find the leading term of the asymptotics of the following integral for $s \rightarrow +\infty$:

$$A(s) = \int_{-\infty}^{\infty} \exp\left[i\left(sz + \frac{z^3}{3}\right)\right] dz.$$

Hints: use the method of Steepest Descents. Find the saddle points of the integrand. Change the integration variable so that the saddle points are stationary. Select the saddle point that produces the main contribution to the integral. Deform the integration contour into the suitable steepest descent paths in the complex plane.

See Fig. 1 for a visual help.



Figure 1: The graphs of u(x, y) = const (black curves) and v(x, y) = const (green curves), where $u(x, y) + iv(x, y) = i\left(z + \frac{z^3}{3}\right)$, z = x + iy. Background color indicates the values of u(x, y)

2. (55 points) Find the leading term of the asymptotics of the following integral for $s \rightarrow +\infty$:

$$B(s) = \int_{-\infty}^{\infty} \exp\left[i\left(sz + \frac{z^3}{3}\right)\right] \frac{z^2}{z^2 + 1} \,\mathrm{d}z.$$

Hints: use the approach of Problem 1. Take into account the contribution to the integral due to the pole of the integrand.