Show all your work and indicate your reasoning in order to receive the credit. Present your answers in *low-entropy* form. Write your name on the problems page and enclose it together with your solutions. Use **only** the methods we introduced in class.

Name: _____

Date: _____

Question:	1	2	3	4	5	6	Total
Points:	10	15	30	10	15	20	100
Score:							

The problem:

Find the frequencies of small oscillations of a uniform flexible unextendable chain of length L that is fixed at the upper end and free at the lower end. (The acceleration of gravity is g.)

For the mathematical description of the problem, consider the course notes *Oscillations of a hanging chain*.

To check whether your theory make sense, observe the frequency of oscillations of a simple pendulum with the lowest frequency of oscillations of a chain of the same length and compare your measurements with your predictions. (The chain and the pendulum are provided by the instructor.)

1. (10 points) Write the equation for horizontal displacements of the chain in in dimensionless form. Hint: as a result you have all coefficients in the equation either zeroes or ones.
2. (15 points) Obtain the solution of the equation as a contour integral in the complex plane. Hint: chose a close contour around the origin; the solution you obtain this way automatically conforms to the boundary condition at the free end of the chain. Also, please notice that the integrand has an essential singularity at the origin.
3. Use the method of stationary phase to find an analytical approximation for the solution.
   (a) (10 points) Convert the moving stationary point in the integrand to a fixed one.

(b) (10 points) Deform the integration contour to the unit circle. Next, change your integration variable from a complex integration variable (t) to a real integration variable (θ): $t = e^{i\theta}$, $0 \le \theta \le 2\pi$.

(c) (10 points) Obtain the asymptotics of the solution.

4. (10 points) Use the boundary condition at the fixed end of the chain to find the expression for the chain frequencies.

5. (15 points) Compare your prediction for the lowest frequency of the oscillations to the actual frequency of the sample chain and to the frequency of a simple pendulum of the same length.

6. (20 points) Provide a clear *self-contained* description of the problem you solved and the solution steps.