THINGS TO KNOW FOR MIDTERM I

SPRING SEMESTER 2017

http://www.phys.uconn.edu/~rozman/Courses/P2400_17S/

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Gamma function:

$$\Gamma(x) = \int_0^\infty e^{-t}\, t^{x-1}\, \mathrm{d}t$$

$$\Gamma(x+1) = x\Gamma(x)$$

$$\Gamma(n) = (n-1)! \quad n = 1, 2, 3, \ldots$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}, \quad \Gamma(1) = 1$$

Beta function:

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, \mathrm{d}t$$

$$B(x,y) = B(y,x)$$

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

Leibniz's formula:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\int_{a(x)}^{b(x)} f(t,x)\,\mathrm{d}t\right\} = \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(t,x)\,\mathrm{d}t + f(b(x),x)\frac{\mathrm{d}b}{\mathrm{d}x} - f(a(x),x)\frac{\mathrm{d}a}{\mathrm{d}x}$$

Euler's formula:

$$e^{ix} = \cos(x) + i\sin(x), \quad \text{where} \quad i \equiv \sqrt{-1}$$

$$\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

$$\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$$

$$e^{\pm i\pi} = -1, \qquad e^{\pm i\frac{\pi}{2}} = \pm i, \qquad e^{2\pi i n} = 1, \quad n \in \mathbb{Z}$$

Complex numbers – coordinate and polar form:

$$z = x + iy = re^{i\varphi}, \quad \text{where} \quad i \equiv \sqrt{-1} = e^{i\frac{\pi}{2}}$$

$$r \equiv |z| = \sqrt{x^2+y^2}, \quad \tan\varphi = \frac{y}{x}, \quad \tan\frac{\varphi}{2} = \frac{y}{x+r} \quad \longleftrightarrow \quad x = r\cos\varphi, \quad y = r\sin\varphi$$

Cauchy-Riemann equations:

$$f(z) = u(x,y) + iv(x,y)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

Cauchy's integral theorem:

$$\oint_C f(z)\,\mathrm{d}z = 0,$$

if $f(z)$ is analytic inside a closed contour C. Alternatively,

$$\int_{L_1} f(z)\,\mathrm{d}z = \int_{L_2} f(z)\,\mathrm{d}z,$$

where L_1 and L_2 are two different complex paths that share the start and the end points.

Laurent series:

$$f(z) = \frac{a_{-m}}{(z-z_0)^m} + \frac{a_{-m+1}}{(z-z_0)^{m-1}} + \ldots + \frac{a_{-1}}{(z-z_0)} + a_0 + a_1(z-z_0) + a_2(z-z_0)^2 + a_3(z-z_0)^3 + \ldots,$$

$$f(z) = \sum_{n=-m}^{\infty} a_n (z-z_0)^n.$$

$f(z)$ has an isolated singularity at z_0; m - the order of the pole.

$$a_n = \frac{1}{(m+n)!} \frac{\mathrm{d}^{m+n}}{\mathrm{d}z^{m+n}} \left[(z-z_0)^m f(z)\right]\Big|_{z=z_0}.$$

Residues:

$$\oint_C f(z)\,\mathrm{d}z = 2\pi i a_{-1},$$

where C is a closed contour that includes z_0, the single singularity of $f(z)$.

$$\mathrm{Res}(f(z), z = z_0) \equiv a_{-1}.$$

For a simple pole,

$$\mathrm{Res}(f(z), z = z_0) = \lim_{z \to z_0} \left[(z-z_0) f(z)\right].$$

If $f(z) = \frac{p(z)}{q(z)}$ then $q(z_0) = 0$ and

$$\mathrm{Res}(f(z), z = z_0) = \frac{p(z_0)}{q'(z_0)},$$

where $q' = \frac{\mathrm{d}q}{\mathrm{d}z}$.

For a pole of order m,

$$\mathrm{Res}(f(z), z = z_0) = \lim_{z \to z_0} \frac{1}{(m-1)!} \frac{\mathrm{d}^{m-1}}{\mathrm{d}z^{m-1}} \left[(z-z_0)^m f(z)\right].$$

Jordan's lemma: yields a simple way to calculate the integral along the real axis of functions $f(z) = e^{iaz} g(z)$.