Things to know for Midterm I

Gamma function:

$$\Gamma(x) = \int_0^\infty e^{-t}\, t^{x-1}\, \mathrm{d}t$$

$$\Gamma(x+1) = x\,\Gamma(x)$$

$$\Gamma(n) = (n-1)! \quad n = 1, 2, 3, \ldots$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}, \quad \Gamma(1) = 1$$

Beta function:

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, \mathrm{d}t$$

$$B(x,y) = B(y,x)$$

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

Useful tools:

- $$\int_{-\infty}^{\infty} f\left(x - \frac{1}{x}\right)\, \mathrm{d}x = \int_{-\infty}^{\infty} f(x)\, \mathrm{d}x$$

- Frullani's integral:

$$\int_0^\infty \frac{f(ax) - f(bx)}{x}\, \mathrm{d}x = [f(\infty) - f(0)]\, \ln\frac{a}{b}$$

- Feynman parametrization:

$$\frac{1}{ab} = \int_0^1 \frac{\mathrm{d}x}{[ax + b(1-x)]^2}$$

- Schwinger parametrization:

$$\frac{1}{\sqrt{R}} = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-Ru^2}\, \mathrm{d}u, \qquad \frac{1}{R^\alpha} = \frac{1}{\Gamma(\alpha)} \int_0^\infty e^{-Rt}\, t^{\alpha-1}\, \mathrm{d}t$$

Leibniz's formula:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\int\limits_{a(x)}^{b(x)} f(t,x)\,\mathrm{d}t\right\} = \int\limits_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(t,x)\,\mathrm{d}t + f(b(x),x)\,\frac{\mathrm{d}b}{\mathrm{d}x} - f(a(x),x)\,\frac{\mathrm{d}a}{\mathrm{d}x}$$

Euler's formula:

$$e^{ix} = \cos(x) + i\sin(x), \quad \text{where} \quad i \equiv \sqrt{-1}$$

$$\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

$$\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$$

$$e^{\pm i\pi} = -1, \qquad e^{\pm i\frac{\pi}{2}} = \pm i, \qquad e^{2\pi i n} = 1, \quad n \in \mathbb{Z}$$

Complex numbers – coordinate and polar form:

$$z = x + iy = re^{i\varphi}, \quad \text{where} \quad i \equiv \sqrt{-1} = e^{i\frac{\pi}{2}}$$

$$r \equiv |z| = \sqrt{x^2 + y^2}, \quad \tan\varphi = \frac{y}{x} \qquad \longleftrightarrow \qquad x = r\cos\varphi, \quad y = r\sin\varphi$$

Cauchy-Riemann equations:

$$f(z) = u(x,y) + iv(x,y)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Cauchy's integral theorem:

$$\oint\limits_C f(z)\,\mathrm{d}z = 0,$$

if $f(z)$ is analytic inside an arbitrary closed contour C. Alternatively,

$$\int\limits_{L_1} f(z)\,\mathrm{d}z = \int\limits_{L_2} f(z)\,\mathrm{d}z,$$

where L_1 and L_2 are two different complex paths that share the start and the end points.