Show all your work and indicate your reasoning in order to receive the credit. Present your answers in *low-entropy* form. Write your name on the problems page and enclose it together with your solutions. Enclose printouts of computer algebra sessions, where relevant.

Name: ______________________________

Date: ______________________________

Question:	1	2	Total
Points:	20	20	40
Score:			

Useful tricks.

1. Assuming that the integrals below exist,

$$\int_{-\infty}^{\infty} f\left(x - \frac{1}{x}\right)\,\mathrm{d}x = \int_{-\infty}^{\infty} f(x)\,\mathrm{d}x. \qquad (A)$$

   (a) (15 points) Use Eq. (A) to evaluate the following integral:

$$\int_{-\infty}^{\infty} e^{-\left(x^2-2\right)^2-\left(\frac{1}{x^2}-2\right)^2}\,\mathrm{d}x$$

   Express your result via gamma function.

   (b) (1 point) Try to evaluate the integral above using a computer algebra system

   (c) (4 points) Use a computer algebra system and compare the numerical values of the integral and your answer.

   bonusbonusbonus

   (d) (bonus points) Prove Eq. (A)

2. The so called *Schwinger parametrization* is based on identities like the following:

$$\frac{1}{\sqrt{R}} = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-Ru^2}\,\mathrm{d}u \qquad (B)$$

   (a) (15 points) The first octant of a three-dimensional coordinate system is filled with continuously distributed charge of density

$$\rho(x, y, z) = \rho_0 e^{-(x^2+2y^2+2z^2)}.$$

Use Eq. (B) to evaluate the electrostatic potential in the origin, which is given by the following integral:

$$V_0 = \frac{\rho_0}{4\pi\epsilon_0} \iiint\limits_{0}^{\infty} \frac{e^{-(x^2+2y^2+2z^2)}}{\sqrt{x^2+y^2+z^2}}\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z. \qquad (C)$$

Hint: The Schwinger parametrization converts the triple integral Eq. (C) into a quadrupple integral. Three out four integrals are gaussian ones, thus can be easily evaluated. Use a computer algebra system to evaluate the last one-dimensional integral that you obtain.

(b) (1 point) Try to evaluate the integral above using a computer algebra system.

(c) (4 points) Use a computer algebra system and compare the numerical values of the integral (without the dimensional factor in front) and your answer.