Name: ______________________________

Date: ______________________________

Show all your work and indicate your reasoning in order to receive the most credit. Present your answers in *low-entropy* form. Write your name on the problems page and staple it together with your solutions.

Hint: to plot the numerical value of an integral vs. a parameter, to compare it with your asymptotics, you may use a similar commands:

```
f[lam_] := NIntegrate[Log[1 + t]*Cos[lam*t], {t,0,1} ]
Plot[f[lam], {lam, 5, 25}]
```

Question:	1	2	3	4	Total
Points:	20	20	15	25	80
Score:					

Integration of rapidly oscillating functions

1. (20 points) Use integration by parts to find the first **two** terms of the approximation as $\lambda \to \infty$ for the following integral:

$$I(\lambda) = \int_0^1 \log(1+t)\, \cos(\lambda t)\, \mathrm{d}t.$$

On the same graph plot the numerical value of the integral, your first approximation, and your second order approximation vs. λ for $5 < \lambda < 25$.

2. (20 points) Use integration by parts to find the first **two** terms of the approximation as $\lambda \to \infty$ for the following integral:

$$I(\lambda) = \int_1^2 \frac{\sin\left(\lambda t^2\right)}{t}\, \mathrm{d}t.$$

On the same graph plot the numerical value of the integral, your first approximation, and your second order approximation vs. λ for $5 < \lambda < 25$.

Method of stationary phase.

3. (15 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int\limits_0^1 \cos\left(\lambda x^2\right) \cosh\left(x^2 + \sin x\right) \mathrm{d}x.$$

On the same graph plot the numerical value of the integral and your approximation vs. λ for $5 < \lambda < 25$.

4. (25 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^1 \cos\left(\lambda x^4\right)\left(1+x^2\right)^{\frac{3}{2}}\,\mathrm{d}x.$$

On the same graph plot the numerical value of the integral and your approximation vs. λ for $5 < \lambda < 25$.

Hint: to evaluate the integral

$$\int_0^\infty e^{i\lambda x^4}\,\mathrm{d}x,$$

use the integration contour shown in Fig. 1 and consider the integral of $e^{i\lambda z^4}$.

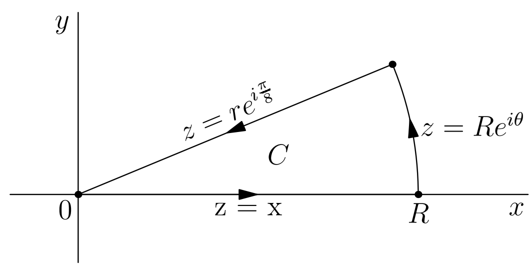

Figure 1: Integration contour for Problem 4. ($R \to \infty$) .