Name: ____________________

Date: ____________________

Show all your work and indicate your reasoning in order to receive the most credit. Present your answers in *low-entropy* form. Write your name on the problems page and staple it together with your solutions.

Hint: to plot the numerical value of an integral, e.g. the integral from Problem 1, to compare it with your asymptotics, you may use the following commands:

```
f[lam_] := NIntegrate[Exp[-lam*(Sinh[x])^2], {x,0,2} ]
Plot[f[lam], {lam, 5, 25}]
```

Question:	1	2	3	4	5	Total
Points:	12	12	12	17	22	75
Score:						

Laplace method for integrals

1. (12 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^2 e^{-\lambda \sinh^2 x} \mathrm{d}x.$$

On the same graph plot the numerical value of the integral and your approximation vs. λ for $5 < \lambda < 25$.

2. (12 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^3 \frac{e^{-\lambda x}}{\cosh x} \mathrm{d}x.$$

On the same graph plot the numerical value of the integral and your approximation vs. λ for $5 < \lambda < 25$.

3. (12 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_0^1 e^{-\lambda \tan x} \mathrm{d}x.$$

On the same graph plot the numerical value of the integral and your approximation vs. λ for $5 < \lambda < 25$.

4. (17 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_{-1}^{1} e^{-(\lambda \sin x)^4} \mathrm{d}x.$$

Express your answer using Gamma function.

On the same graph plot the numerical value of the integral and your approximation vs. λ for $2 < \lambda < 10$.

5. (22 points) Find the leading term of the asymptotics of the following integral for $\lambda \to \infty$:

$$I(\lambda) = \int_{0}^{\infty} e^{-\lambda x - \frac{4}{x^2}} \mathrm{d}x.$$

On the same graph plot the numerical value of the integral and your approximation vs. λ for $3 < \lambda < 7$.

Hint: you need to re-scale the integration variable, similar to what we did in class when we analyzed the asymptotics of Gamma function.