Name: _____

Date: _____

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	10	25	75
Score:							

Computer algebra

1. (10 points)

   ☐ I've installed and tested RuBI - Rule Based Integrator for Mathematica.

Sign and date here: _____

Complex numbers

2. (10 points) Find the coordinate and the polar form of the following complex number:

$$Z = \left(\frac{\sqrt{2} - i\sqrt{2}}{1 - i\sqrt{3}} \right)^{26}.$$

   Answer: $Z = e^{i\frac{\pi}{6}} = \frac{\sqrt{3}}{2} + \frac{i}{2}$

3. (10 points) Find the values of $Z = \left(\sqrt{i}\right)^{i}$.

   Answer: $Z = e^{-\frac{\pi}{4} - \pi n}$, where $n = 0, \pm 1, \pm 2, \ldots$

4. (10 points) Find the coordinate and the polar forms of the solutions of the equation:

$$z^4 = \sqrt{3} - i.$$

   How many roots are there?

Cauchy-Riemann equations

5. (10 points) Use Cauchy-Riemann equations to find the analytic function $f(z)$, $z = x + iy$, such that its real part is as following:

$$\mathrm{Re}\, f(z) = u(x, y) = e^x \sin y,$$

   and

$$f(i\pi) = 0.$$

Express the result for $f(z)$ as a **function of z only.**

Answer: $f(z) = -i\,(e^z + 1)$.

The Cauchy integral theorem

6. (25 points) Evaluate the integral

$$I = \int_0^\infty \sin\left(x^3\right) dx$$

Hints: consider the integral

$$\oint_C e^{-z^3} dz$$

along the contour C sketched in Fig. 1; use the Euler formula; use the fact that

$$\int_0^\infty e^{-x^3} dx \equiv \Gamma\left(\frac{4}{3}\right),$$

where Γ is gamma function. (Can you show this?)

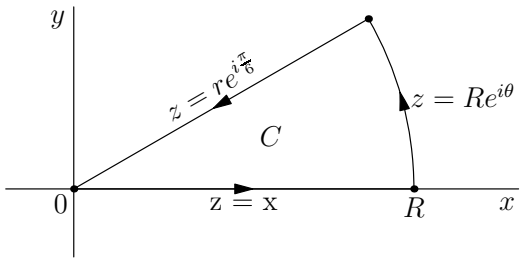

Figure 1: Integration contour for Problem 6. $(R \to \infty)$.

Answer: $I = \frac{1}{2}\,\Gamma\left(\frac{4}{3}\right)$.