Name: _____

Date: _____

Question:	1	2	3	4	5	Total
Points:	5	15	30	5	5	60
Score:						

Frullani's formula

1. (5 points) Evaluate the following integral:

$$I(a,b) = \int_0^\infty \frac{1}{x}\left(\frac{1}{(1+ax)^2} - \frac{1}{(1+bx)^2}\right) \mathrm{d}x,$$

where $a, b > 0$.

Answer: $I(a,b) = \ln \frac{b}{a}$

Leibniz' rule

2. (15 points) Find the positive value of x that maximizes the value of the following integral

$$I(x) = \int_{x-1}^{x+1} \frac{\mathrm{d}u}{\Gamma(u)}.$$

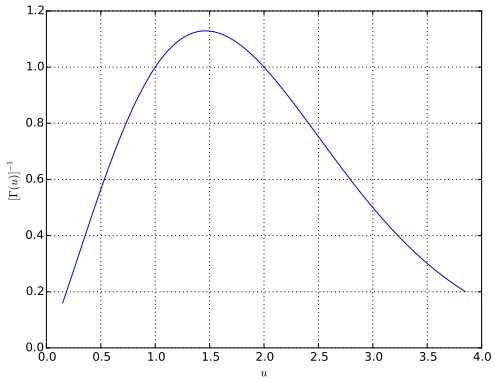

Figure 1: Problem 2: graph of $1/\Gamma(u)$.

Hints: Find the derivative of $I(x)$. What is its value at the maximum of $I(x)$? Simplify the equation that you obtained using the relation $\Gamma(x+1) = x\Gamma(x)$. Solve the equation and select the correct solution.

Answer: $x = \frac{1+\sqrt{5}}{2}$

Differentiation with respect to a parameter

3. (30 points) Evaluate the following integral

$$I = \int_0^\infty e^{-\left(x^2+\frac{1}{x^2}\right)}\,\mathrm{d}x$$

Hints: consider the integral $I(\alpha) = \int_0^\infty e^{-\left(x^2+\frac{\alpha}{x^2}\right)}\mathrm{d}x$. What you need is $I(1)$ and what you know is $I(0)$. Take the derivative of $I(\alpha)$ and transform the integral by introducing the new integration variable $u = \frac{\sqrt{\alpha}}{x}$. Solve the differential equation you thus obtained.

Answer: $I = \frac{\sqrt{\pi}}{2}e^{-2}$

Computer algebra

4. (5 points) Use a computer algebra system to expand the following function, $f(x)$, into a power series about $x = 0$. Keep the terms up to $\sim x^3$.

$$f(x) = e^{\sqrt{\sin(x)}}$$

Print your mathematica session and attach the printout to the rest of your homework.

5. (5 points) Use a computer algebra system to evaluate the following indefinite integral:

$$\int \frac{x}{1+\sin x}\,\mathrm{d}x$$

Print your mathematica session and attach the printout to the rest of your homework.