Name: _____

Date: _____

Question:	1	2	3	4	Total
Points:	10	15	25	15	65
Score:					

Course logistics.

1. (10 points)

   ☐ I've created a JuliaBox account and installed the packages for graphics and symbolic calculations

Sign and date here: _____

Gamma and Beta functions

2. Evaluate the following expressions. Here Γ and B are Euler gamma and beta functions. Only the values of $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ and $\Gamma(1) = 1$ are known.

   (a) (5 points) $\Gamma\left(\frac{5}{2}\right)$

   (b) (5 points) $\Gamma(5)$

   (c) (5 points) $B\left(\frac{1}{2}, \frac{5}{2}\right)$

Integral multiplication trick: the Fresnel integrals

3. (25 points) Evaluate the following integrals

$$C = \int_0^\infty \cos\left(x^2\right) \mathrm{d}x$$

and

$$S = \int_0^\infty \sin\left(x^2\right) \mathrm{d}x.$$

The integrals C and S are named after the Fresnel (French physicist, 17881827). Despite the name, they were first evaluated by Euler (in 1781).

Hint: use Euler formula to write the integral for $F = C + iS$. Square the integral and evaluate it in polar coordinates. Temporary add a convergence factor.

Answer: $C = \sqrt{\frac{\pi}{8}}$, $S = \sqrt{\frac{\pi}{8}}$

Computer algebra

4. (15 points) Use Mathematica to obtain an analytical expression for the following integral:

$$f(x) = \int_{\frac{1}{2}}^{\pi-\frac{1}{2}} \sin^x(y)\,\mathrm{d}y.$$

Plot on the same graph, for $10 \leq x \leq 50$, your result and the approximation to the integral (that we are going to derive later in the course).

$$g(x) = \sqrt{\frac{2\pi}{x}}.$$

Print your mathematica session and attach the printout to the rest of your homework.

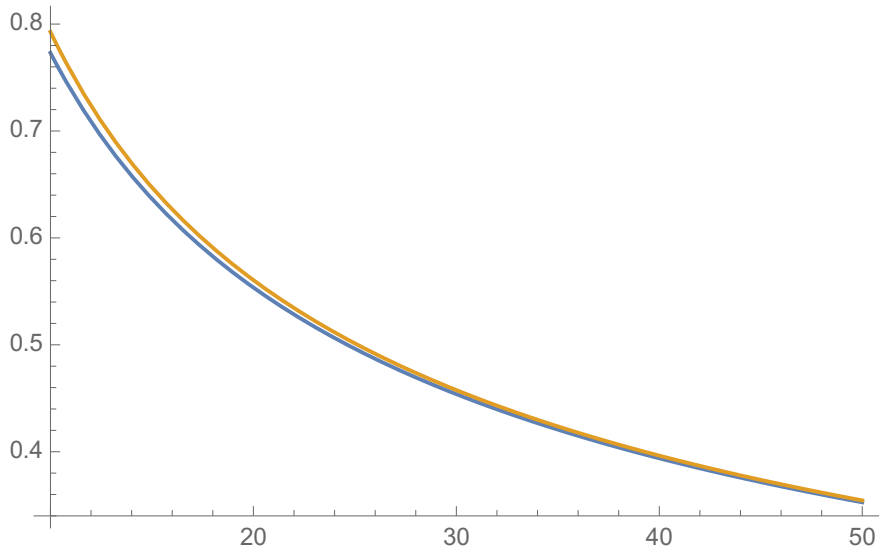

Figure 1: Problem 4: expected graph