8.5.1 $y'' + y' - 2y = 0$

Search the solution in the following form: $y = Ae^{\lambda x}$; $y' = \lambda A e^{\lambda x}$; $y'' = \lambda^2 A e^{\lambda x}$

$$\lambda^2 \cancel{Ae^{\lambda x}} + \lambda \cancel{Ae^{\lambda x}} - 2\cancel{Ae^{\lambda x}} = 0$$

$$\lambda^2 + \lambda - 2 = 0; \quad \lambda_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2} = -\frac{1}{2} \pm \frac{3}{2}$$

$$\lambda_1 = 1; \quad \lambda_2 = -2 \quad \boxed{y = A_1 e^{x} + A_2 e^{-2x}}$$

A_1, A_2 – determined by initial/boundary cond.

12.1.7. $x^2 y'' - 3xy' + 3y = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n; \qquad y' = \sum_{n=0}^{\infty} n a_n x^{n-1}$$

$$xy' = \sum_{n=0}^{\infty} n a_n x^n$$

$$y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2}; \qquad x^2 y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^n$$

$$\sum_{n=0}^{\infty} \underbrace{\left[n(n-1)a_n \underbrace{-3n a_n + 3a_n}_{-3(n-1)a_n}\right]} x^n = 0$$

$$(n-1)(n-3)a_n = 0 \quad \rightarrow \quad a_1 \neq 0, \; a_3 \neq 0$$

$$a_{0,2,4,5\ldots} = 0$$

$$\boxed{y = a_1 x + a_3 x^3}$$

13.2.6 $T = \frac{400}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} e^{-\frac{\pi y}{10}(2n-1)} \underbrace{\sin \frac{\pi x}{10}(2n-1)}_{\operatorname{Im} e^{\frac{i\pi x}{10}(2n-1)}}$

$$= \frac{400}{\pi} \operatorname{Im} \underbrace{\sum_{n=1}^{\infty} \frac{1}{2n-1} e^{\frac{\pi}{10}(2n-1)(ix-y)}}_{\frac{1}{2}\ln \frac{1+e^{\frac{\pi}{10}(ix-y)}}{1-e^{\frac{\pi}{10}(ix-y)}}}$$

$$T = \frac{200}{\pi} \operatorname{Im}\left[\ln \frac{1+e^{\frac{\pi}{10}(ix-y)}}{1-e^{\frac{\pi}{10}(ix-y)}}\right]$$

$$X + iY = Re^{i\varphi} \qquad \varphi = \arctan \frac{Y}{X}$$

$$\operatorname{Im}(\ln(X+iY)) = \operatorname{Im}(\ln R + i\varphi) = \varphi$$

$$T = \frac{200}{\pi}\varphi\,;$$

$$\frac{1+e^{\frac{\pi}{10}(ix-y)}}{1-e^{\frac{\pi}{10}(ix-y)}} = \frac{\left(1+e^{\frac{\pi}{10}(ix-y)}\right)\left(1-e^{\frac{\pi}{10}(-ix-y)}\right)}{\left(1-e^{\frac{\pi}{10}(ix-y)}\right)\left(1-e^{\frac{\pi}{10}(-ix-y)}\right)} =$$

$$= \frac{1-e^{-\frac{\pi}{5}y} + e^{-\frac{\pi}{10}y}\left(e^{\frac{\pi}{10}ix} - e^{-\frac{\pi}{10}ix}\right)}{(\;\ldots\ldots\ldots\;)} = \frac{\left(1-e^{-\frac{\pi}{5}y}\right) + 2ie^{-\frac{\pi}{10}y}\sin\frac{\pi}{10}x}{(\;\ldots\ldots\ldots\;)}$$

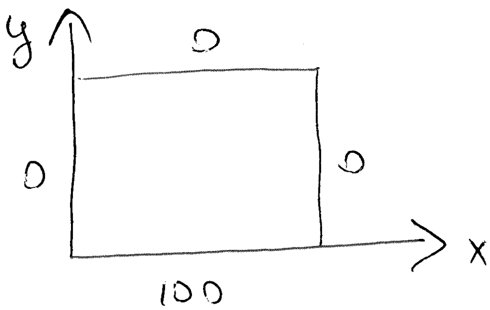
real

$$\varphi = \arctan \frac{2e^{-\frac{\pi}{10}y}\sin\frac{\pi}{10}x}{1-e^{-\frac{\pi}{5}y}} = \arctan \frac{\sin\frac{\pi}{10}x}{\frac{1}{2}\left(e^{\frac{\pi}{10}y}-e^{-\frac{\pi}{10}y}\right)}$$

$$= \arctan \frac{\sin\frac{\pi}{10}x}{\sinh\frac{\pi}{10}y}$$

$$\boxed{T = \frac{200}{\pi}\arctan \frac{\sin\frac{\pi}{10}x}{\sinh\frac{\pi}{10}y}}$$

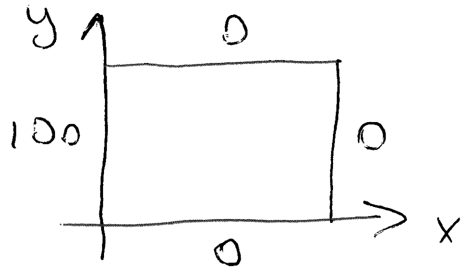
13.2.11



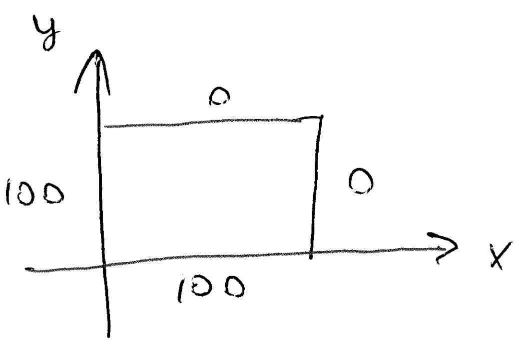
Given solution:

$$T_1 = \sum_{\text{odd } n} \frac{400}{n\pi \sinh n\pi} \times \sinh \frac{n\pi}{10}(10-y) \cdot \sin \frac{n\pi x}{10}$$

Making the swap $x \leftrightarrow y$ can get the solution for the following problem.



$$T_2 = \sum_{\text{odd } n} \frac{400}{n\pi \sinh n\pi} \times \sinh \frac{n\pi}{10}(10-x) \sin \frac{n\pi y}{10}$$



$$\boxed{T = T_1 + T_2}$$