**8.2.5** $xy' - xy = y \qquad y(1) = 1$

Separate variables:

$xy' = y + xy = y(1+x); \quad \frac{dy}{dx} = \frac{1+x}{x} y;$

$\frac{dy}{y} = \left(1 + \frac{1}{x}\right)dx \rightarrow \int \frac{dy}{y} = \int \left(1+\frac{1}{x}\right)dx + C$

$\ln y = x + \ln x + C$

$y = e^x \underbrace{e^{\ln x}}_{x} \underbrace{e^C}_{A} = Axe^x$

Initial condition: $y(1) = Ae = 1 \rightarrow A = \frac{1}{e}$

$y = \frac{1}{e} x e^x = \boxed{x e^{x-1}}$

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**8.2.8** $y' + 2xy^2 = 0 \qquad y(2) = 1$

$\frac{dy}{dx} = -2xy^2 \rightarrow \frac{dy}{y^2} = -2x\,dx$

$\int \frac{dy}{y^2} = -\int 2x\,dx - C \rightarrow -\frac{1}{y} = -x^2 - C$

$y = \frac{1}{x^2+C};$ Initial cond: $y(2) = \frac{1}{4+C} = 1$

$4 + C = 1 \rightarrow C = -3$

$$\boxed{y = \frac{1}{x^2 - 3}}$$

**8 3.4** $\quad 2xy' + y = 2x^{5/2}$

$$y' + \frac{1}{2x}y = x^{3/2}$$ — linear, first orde ODE

$p(x) = \frac{1}{2x}$, $q(x) = x^{3/2}$

$$\int p(x)\,dx = \frac{1}{2}\ln x = \ln x^{1/2}$$

$$e^{\int p(x)\,dx} = x^{1/2}$$

$$\int q(x)\, e^{\int p(x)\,dx'} dx = \int x^{3/2} \cdot x^{1/2}\, dx = \frac{x^3}{3}$$

$$y = \frac{1}{x^{1/2}} \cdot \frac{x^3}{3} + \frac{C}{x^{1/2}} = \boxed{\frac{1}{3}x^{5/2} + Cx^{-1/2}}$$

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8.3.11 $\quad y' + y\cos x = \sin 2x$

$p(x) = \cos x$

$q(x) = \sin 2x =$

$= 2\sin x \cos x$

$$\int p(x)\,dx = \sin(x)$$

$$\int q(x)\, e^{\int p(x)dx} dx = \int 2\sin x\, e^{\sin x}\, d\sin x =$$

$$= 2\int \sin x \cdot d e^{\sin x} = 2\sin x\, e^{\sin x} - 2\int \underbrace{e^{\sin(x)}\, d\sin x}_{d e^{\sin x}}$$

$$= 2\sin x\, e^{\sin x} - 2e^{\sin x} = 2e^{\sin(x)}(\sin x - 1)$$

$$y = e^{-\sin x} \cdot 2e^{\sin x}(\sin x - 1) + Ce^{-\sin x} =$$

$$= \boxed{2(\sin(x) - 1) + Ce^{-\sin(x)}}$$