14.2.58 $\quad u(x,y) = \cosh(y)\cos(x)$

(a)
$$\frac{\partial u}{\partial x} = -\cosh(y)\sin(x)$$
$$\frac{\partial^2 u}{\partial x^2} = -\cosh(y)\cos(x)$$
$$\frac{\partial u}{\partial y} = \sinh(y)\cos(x)$$
$$\frac{\partial^2 u}{\partial y^2} = \cosh(y)\cos(x)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0}$$

(b) Cauchy-Riemann conditions:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \Rightarrow \frac{\partial v}{\partial y} = -\cosh(y)\sin(x)$$
$$\Downarrow$$
$$v(x,y) = -\sinh(y)\sin(x) + p(x)$$

$$-\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} \Rightarrow \frac{\partial v}{\partial x} = -\sinh(y)\cos(x)$$
$$\Downarrow$$
$$v(x,y) = -\sinh(y)\sin(x) + q(y)$$

$$v(x,y) = -\sinh(y)\sin(x) \quad (+\text{ const})$$

(c) $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ — similar to (a)

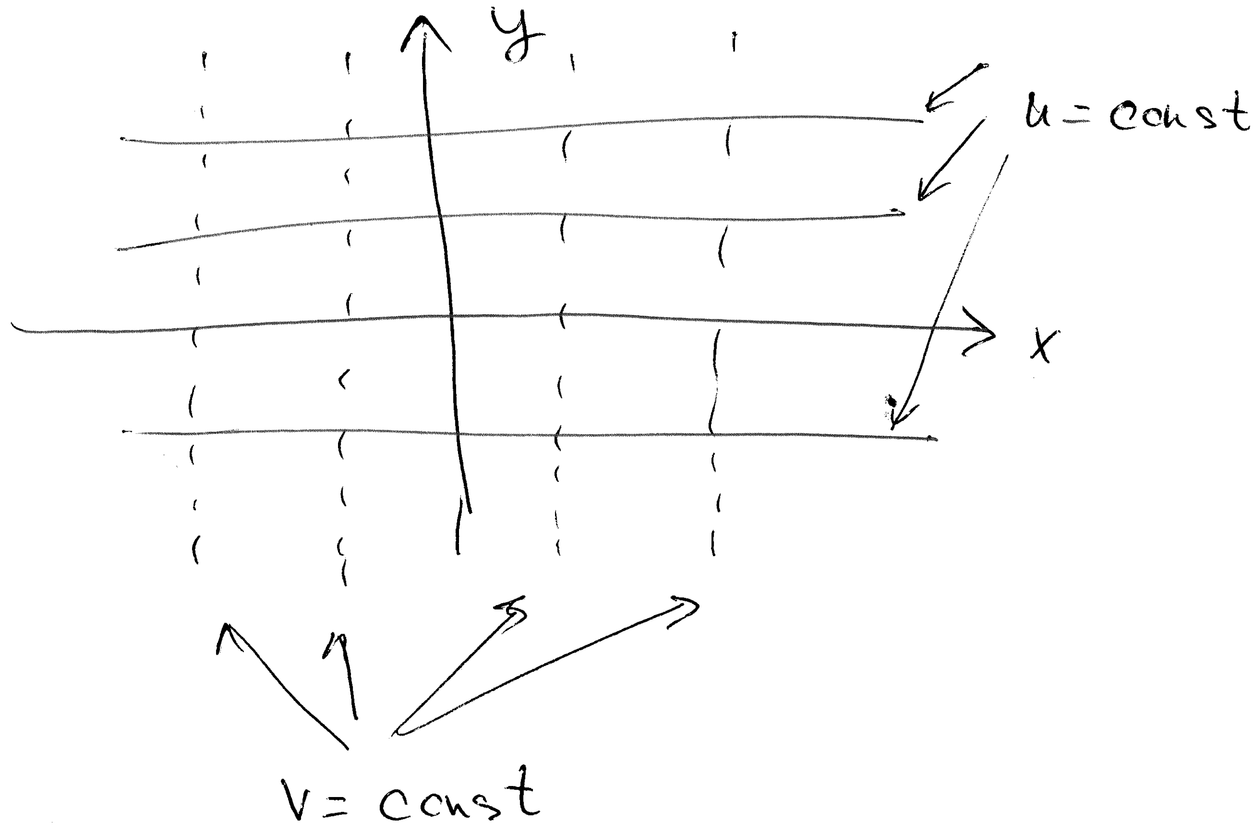
14.9.2

$$w = \frac{1}{2i}(z+1) = \frac{1}{2i}\left(x+1+iy\right)$$

$$= \frac{y}{2}$$ [redacted] $$\frac{1}{2}i\,(x+1)$$

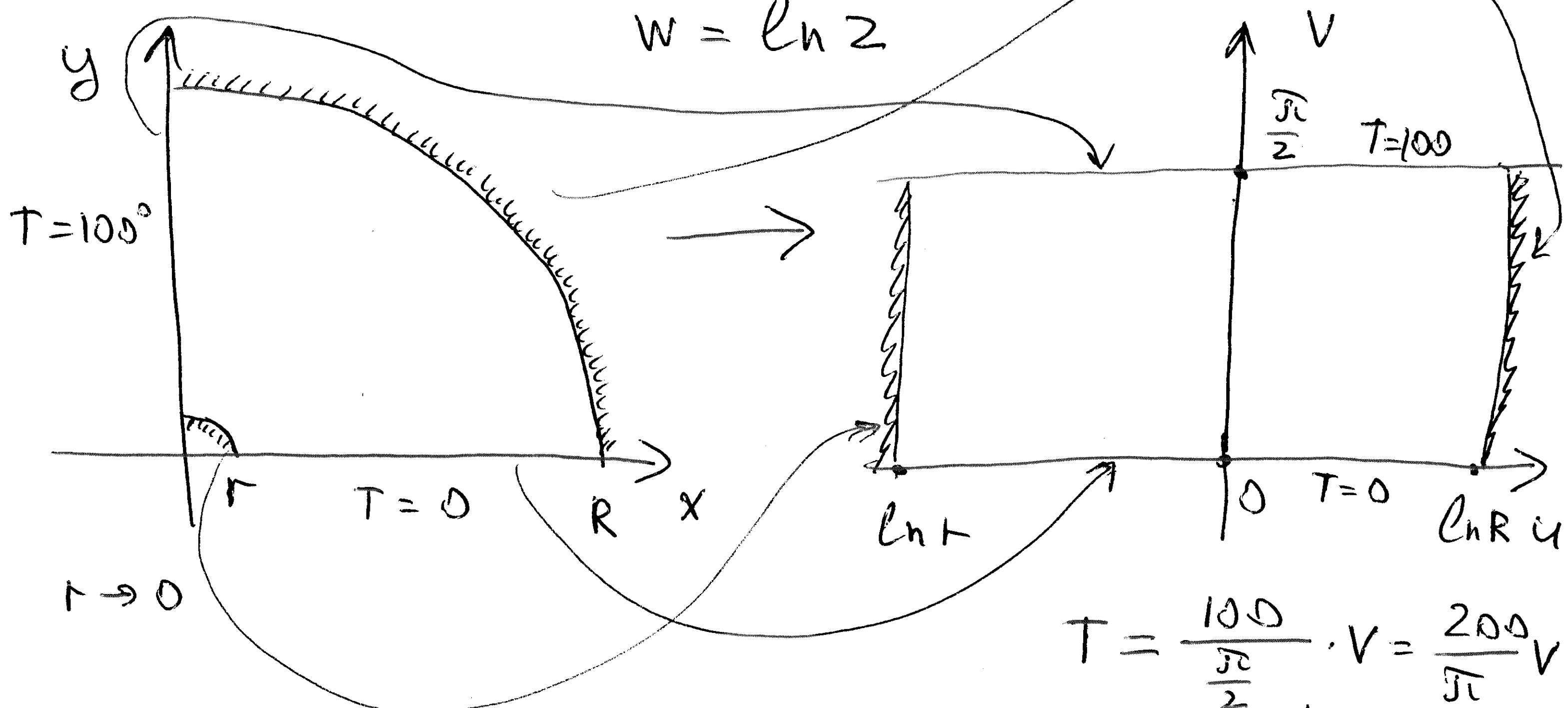
$$u = \mathrm{Re}(w) = \frac{y}{2}$$

$$v = \mathrm{Im}(w) = -\frac{1}{2}(x+1)$$



14.10.4

$$z = x + iy$$
$$w = u + iv$$
$$w = \ln z$$

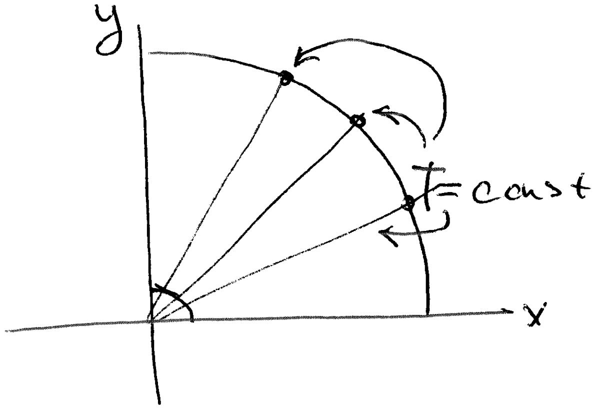


$$T = \frac{100}{\frac{\pi}{2}} \cdot v = \frac{200}{\pi} v$$

$$\left.\begin{array}{l} z = |z| e^{i\theta} \\ \theta = \arctan \frac{y}{x} \end{array}\right\} \quad v \equiv \theta$$

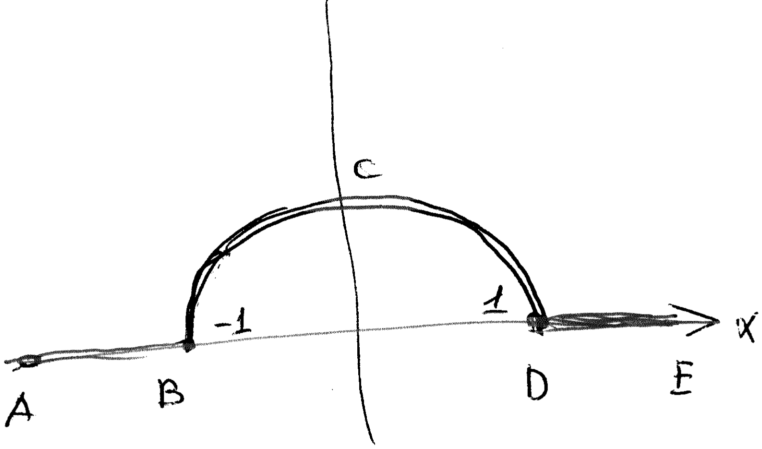
$$T = \frac{200}{\pi} \arctan \frac{y}{x}$$

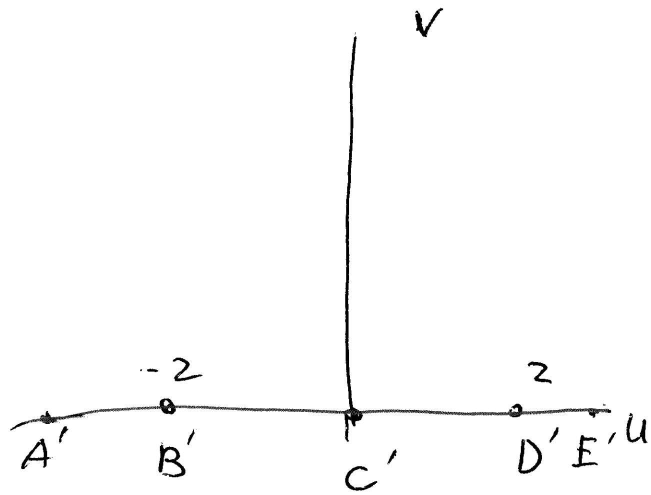
isothermals : $\frac{y}{x} = \text{const} \rightarrow y = \beta x$



14.10.9

$$W = z + \frac{1}{z}$$





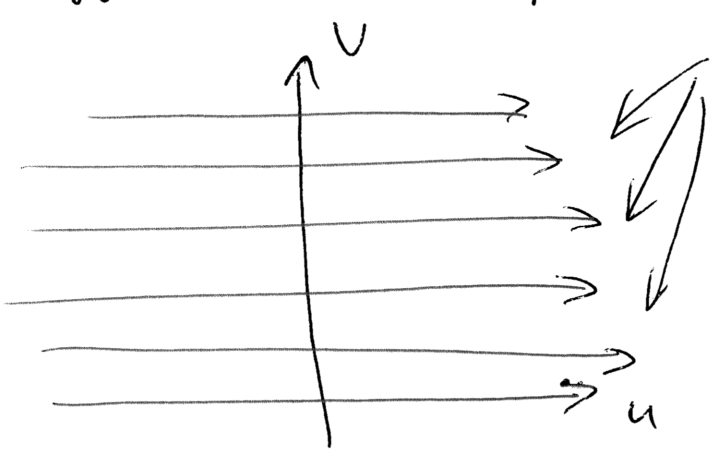
$AB \rightarrow A'B'$

$DE \rightarrow D'E'$

BCD: $z = e^{i\theta}$, $\pi < \theta < 0$, $w = e^{i\theta} + e^{-i\theta} = 2\cos\theta$

$BCD \rightarrow B'C'D'$

"Water" $\rightarrow$ upper complex semi-plane



Streamlines

$V = \text{const}$

$$V = \operatorname{Im}(w) =$$

$$= \operatorname{Im}\left(x + iy + \frac{1}{x+iy}\right)$$

$$= \operatorname{Im}\left(x + iy + \frac{x - iy}{x^2+y^2}\right) = y + \frac{y}{x^2+y^2}$$

Streamlines: $\boxed{y + \frac{y}{x^2+y^2} = \text{const}}$