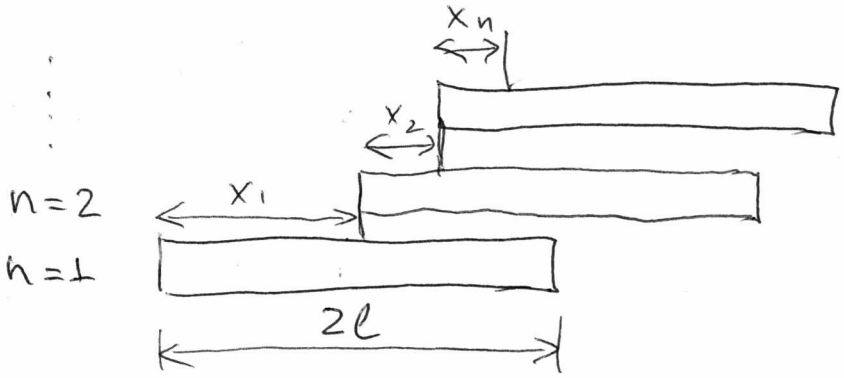
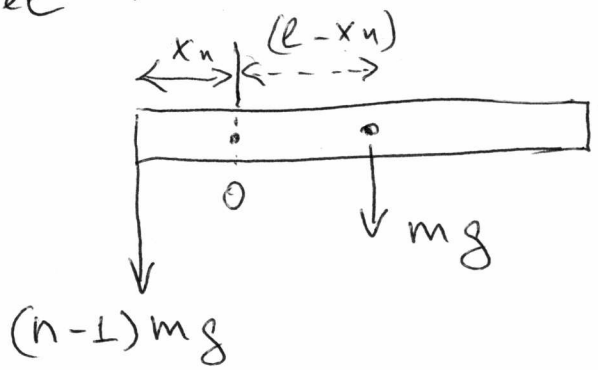
1.16.2

$x_n - ?$



Let us consider the equilibrium of dowel No n:



there are $(n-1)$ dowels below, so the force acting on the dowel's left end is $mg(n-1)$

Balance of torques:

$$(n-1)\,mg \cdot x_n = mg\,(\ell - x_n)$$

$$(n-1)\,x_n = \ell - x_n$$

$$n\,x_n = \ell$$

$$\boxed{x_n = \frac{\ell}{n}}$$

1.16.24 Find the sum

$$\ln 3 + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \dots$$

The given series is "almost" the Taylor series for $\exp(\dots)$, with the zero'th term missing. Thus,

$$\ln 3 + \frac{(\ln 3)^2}{2!} + \dots = e^{\ln 3} - 1 = 3 - 1 = 2$$

1.16.31a Find the number of terms required to find the sum with error $\approx \frac{1}{200}$

$$\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{100}}} = \sum_{n=1}^{N} \frac{1}{n^{1+\frac{1}{100}}} + \sum_{\substack{n=N+1 \\ (N \gg 1)}}^{\infty} \frac{1}{n^{1+\frac{1}{100}}}$$

The second sum is the error we make when we take into account only the first N terms.

$$\sum_{n=N+1}^{\infty} \frac{1}{n^{1+\frac{1}{100}}} \approx \int_{N+1}^{\infty} \frac{dn}{n^{1+\frac{1}{100}}} = -100 \left. \frac{1}{n^{1/100}} \right|_{N+1}^{\infty}$$

$$= \frac{100}{(N+1)^{1/100}} \approx \frac{100}{N^{1/100}} \sim \frac{1}{200}; \quad N^{1/100} \sim 2 \cdot 10^4; \quad N \approx 2^{100} \cdot 10^{400}$$