

#5

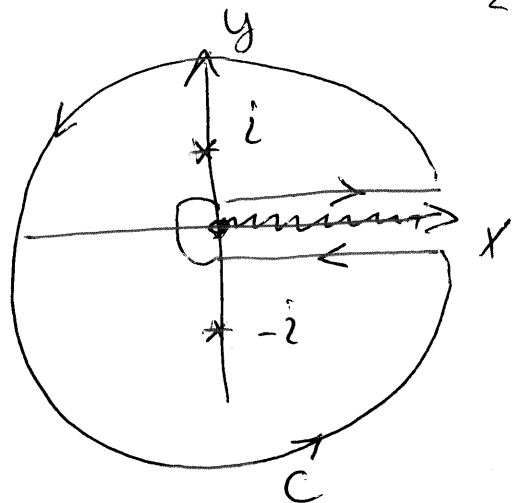
$$I = \int_0^{\infty} \frac{x^{1/3}}{1+x^2} dx$$

$$x \rightarrow e^{2\pi i} x = x$$

$$x^{1/3} \rightarrow e^{\frac{2\pi i}{3}} x$$

$$x^2 \rightarrow e^{4\pi i} x = x$$

$$I \rightarrow e^{\frac{2\pi i}{3}} I$$



$$\oint_C \frac{z^{1/3}}{1+z^2} dz = I + e^{\frac{2\pi i}{3}} I + 0 + 0$$

$$\oint \frac{z^{1/3}}{1+z^2} dz = 2\pi i (\text{Res}(i) + \text{Res}(-i))$$

$$\text{Res}\left(\frac{z^{1/3}}{1+z^2}, z=e^{i\frac{\pi}{2}}\right) = \frac{z^{1/3}}{2z} \Big|_{z=e^{i\frac{\pi}{2}}} = \frac{1}{2} e^{-i\frac{\pi}{2} \cdot \frac{2}{3}} = \frac{1}{2} e^{-i\frac{\pi}{3}}$$

$$\text{Res}\left(\frac{z^{1/3}}{1+z^2}, z=e^{i\frac{3\pi}{2}}\right) = \frac{z^{1/3}}{2z} \Big|_{z=e^{i\frac{3\pi}{2}}} = \frac{1}{2} e^{-\frac{2}{3}i \cdot \frac{3}{2}\pi} = \frac{1}{2} e^{-i\pi} = -\frac{1}{2}$$

$$I(1 - e^{\frac{2\pi i}{3}}) = 2\pi i \cdot \frac{1}{2} (e^{-i\frac{\pi}{3}} - 1)$$

$$I = \pi i \frac{e^{-i\frac{\pi}{3}} - 1}{1 - e^{\frac{2\pi i}{3}}} = \pi i \frac{-1 - e^{i\frac{2\pi}{3}}}{1 - e^{i\frac{2\pi}{3}}} = -\pi i \frac{1 + e^{i\frac{2\pi}{3}}}{1 - e^{i\frac{2\pi}{3}}} = -\pi i \frac{2\cos\frac{\pi}{3}}{-2i\sin\frac{\pi}{3}} = \left| \frac{\pi}{\sqrt{3}} \right|$$