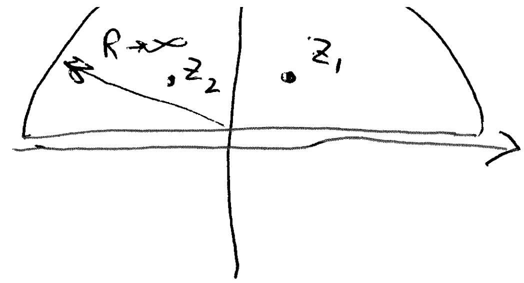
\# 4



$$I = \int_{-\infty} \frac{e^{i\omega x}}{1+x^4}\,dx$$

$$= \oint_C \frac{e^{i\omega z}}{1+z^4}\,dz$$

$$= 2\pi i \sum_{\operatorname{Re} z>0} \operatorname{Res}\left(\frac{e^{i\omega z}}{1+z^4}\right)$$

Since $\omega > 0 \rightarrow$
$i\omega(iy) = -\omega y < 0$
$e^{-\omega y^2} \rightarrow 0$ when $y \rightarrow \infty$

The contribution to the integral from the semicircle in the upper half of the complex plane is negligible

$$1+z^4 = 0; \quad z^4 = -1 = e^{\pi i + 2n i\pi}$$

$$z_1 = e^{i\frac{\pi}{4}}; \quad z_2 = e^{i\frac{3\pi}{4}}, \quad z_3 = e^{i\frac{5\pi}{4}}; \quad z_4 = e^{i\frac{7\pi}{4}}$$

Only z_1 and z_2 are within the integration contour 'C'. $z_1 = e^{i\frac{\pi}{4}} = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$; $z_2 = e^{i\frac{3\pi}{4}} = -\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

$$\operatorname{Res}\left(\frac{e^{i\omega z}}{1+z^4}, z=z_1\right) = \frac{e^{i\omega z_1}}{4z_1^3} = \frac{1}{4}e^{-i\frac{3}{4}\pi}e^{i\omega(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}})}$$

$$= \frac{1}{4}e^{-\frac{\omega}{\sqrt{2}}}e^{i(-\frac{3}{4}\pi+\frac{\omega}{\sqrt{2}})} = -\frac{1}{4}e^{-\frac{\omega}{\sqrt{2}}}e^{i(\ldots}$$

$$\operatorname{Res}\left(\frac{e^{i\omega z}}{1+z^4}, z=z_2\right) = \frac{e^{i\omega z_2}}{4z_2^3} = \frac{1}{4}e^{-i\frac{9\pi}{4}}e^{i\omega(-\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}})}$$

$$= \frac{1}{4}e^{-\frac{\omega}{\sqrt{2}}}e^{i(\frac{\pi}{4}-\frac{\omega}{\sqrt{2}})}$$

$$\operatorname{Res}(z_1)+\operatorname{Res}(z_2) = \frac{1}{4}e^{-\frac{\omega}{\sqrt{2}}}\left[e^{i(-\frac{\pi}{4}-\frac{\omega}{\sqrt{2}})} - e^{i(\frac{\pi}{4}+\frac{\omega}{\sqrt{2}})}\right]$$

$$\boxed{I = \pi e^{-\frac{\omega}{\sqrt{2}}}\sin\left(\frac{\pi}{4}+\frac{\omega}{\sqrt{2}}\right)} = -\frac{i}{2}e^{-\frac{\omega}{\sqrt{2}}}\sin\left(\frac{\pi}{4}+\frac{\omega}{\sqrt{2}}\right)$$