HW6

due October 22, 2025

Name: ______________________________

Date: ______________________________

Collaborators: ____

(Collaborators submit their individually written assignments together, in class, in person)

GitHub username: __________________________

Question:	1	2	3	4	Total
Points:	5	30	35	5	75
Score:					

Instructor/grader comments:

1. (5 points) I watched the video How Fast Are Computers (in Human Terms)? which is a part of Homework 06 assignment.

   Sign and date here: ______________________________

2. (30 points) **Moore's law**

   Moore's law is the observation that the number of transistors in an integrated circuit doubles about every two years.

   The observation is named after Gordon Moore, an American businessman, engineer, and the co-founder of Intel. In an article published in 1965, Moore observed that the number of components (transistors, resistors, diodes, or capacitors) in a dense integrated circuit had doubled approximately every year and speculated that it would continue to do so for at least the next ten years. In 1975, looking forward to the next decade, he revised the forecast to a still exponential growth but with a lower growth rate. While Moore did not use empirical evidence in forecasting that the historical trend would continue, his prediction has held since 1975 and has since become known as a "law".

   Moore's prediction has been used in the semiconductor industry to guide long-term planning and to set targets for research and development, thus functioning to some extent as a self-fulfilling prophecy.

   At some point, Moore's law will stop because we will no longer be able to shrink the spaces between components on a chip. While we're not certain where the limits are, chip pathways can't be shorter than a single molecule, and the actual physical limit is likely larger than that. Industry experts have not reached a consensus on exactly when Moore's law will cease to apply.

   The goal of this assignment is to determine the doubling time in the Moore's law, T_m,

   $$N(t) \sim 2^{t/T_m}, \tag{1}$$

   from the observational data on the number of transistors in an integrated circuit vs time in years.

   (a) Run the provided Julia code that downloads the number of transistors in an integrated circuits vs. the year of manufacture, and load the data into a DataFrame.

The dataframe that you create contains two columns: the year of manufacture ("Year") and the number of components ("Num").

Plot the data is `semilogy` axes and observe that the grows of the number of transistors vs. time is indeed close to the exponential one, i.e. the graph of the dependency $N(t)$ is very close to a linear one when plotted in `semilogy` axes.

(b) Use the function `linear_regression()` that we wrote in class to find the best linear fit of the data in the coordinates $(Year, \log(Num))$. In the same figure plot the observational data as disconnected symbols and the best fit line as a dashed line. Use the proper coordinate system that clearly demonstrate the exponential grows. Provide the axis labels, grid, title, legend.

The expected plot is shown in Fig. 1.

(c) The slope of the best linear fit, β that you obtained, gives you the dependence of the number of transistors vs. time in the form

$$N(t) \sim e^{\beta t}. \tag{2}$$

Find the doubling period T_m, as defined in Eq. (1). Explain your reasoning. When providing the numerical value of T_m, round it to the meaningful number of digits.

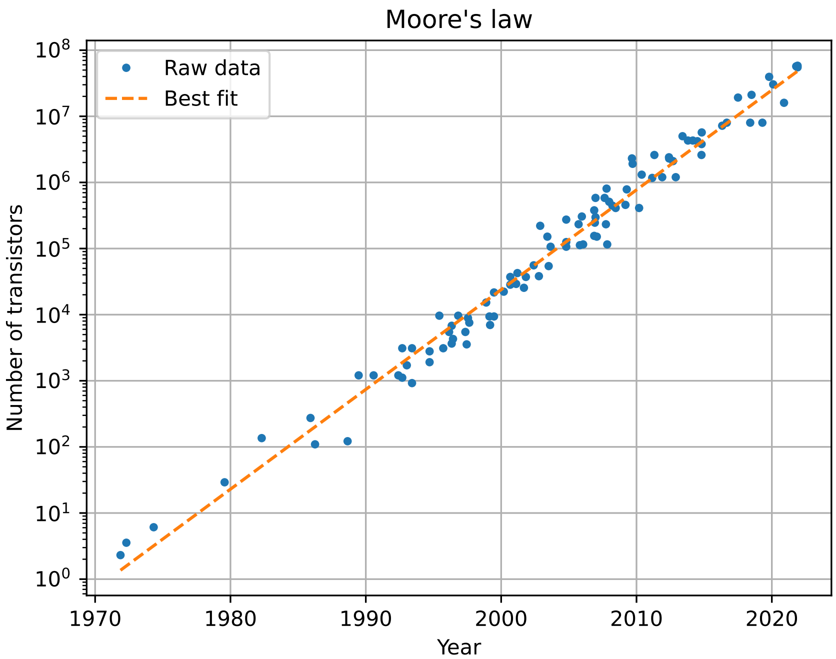

Figure 1: Expected plot for Problem 2(b).

3. (35 points) **Fermi acceleration**

   Fermi acceleration is a mechanism, first suggested by Enrico Fermi in 1949, to explain acceleration of particles in cosmic rays. Fermi studied charged particles being reflected by the moving interstellar magnetic field and either gaining or losing energy, depending on whether the ”magnetic mirror” is approaching or receding. In a typical environment, Fermi argued, the probability of a head-on collision is greater than a head-tail collision, so particles would, on average, be accelerated. Since then Fermi acceleration has been used to explain a number of natural phenomena and several simple mathematical models demonstrating Fermi acceleration have been proposed.

   One of a simple mathematical models proposed to describe the Fermi acceleration is the so-called The Fermi-Ulam model. Consider a particle that moves vertically in the uniform gravitational field with the acceleration of gravity g. The particle elastically collides with a harmonically oscillating floor and the stationary ceiling. The floor and the ceiling represent the magnetic mirrors which reflect the cosmic particles.

   (a) Complete the code provide in the assignment.

   (b) Follow the motion of the ball for `tspan = (0.0, 100.0)`. Plot the trajectory of the ball as well as the position of the floor. Provide the axes labels, grids, title, legend

   The expected plot is shown in Fig. 2.

   (c) Follow the motion of the particle for `tspan = (0.0, 1000.0)`. Plot energy of the ball as a function of time.. Provide the axes labels, grid, title.

   The expected plot is shown in Fig. 3.

   Does the particle monotonously gains energy? Could the Fermi-Ulam model be used to explain observed acceleration of charged particles in cosmic rays? Present your observations and the conclusion.

4. (5 points) **Submission of the assignment.**

   Clear all output cells in your Jupyter notebook(s), then save and close it. Delete any unnecessary or temporary notebooks you created (e.g., `Untitled.ipynb`). Commit all your code changes, making sure to include updates to Project.toml and Manifest.toml. Finally, push your commits to the assignment's GitHub repository.

   The submitted code must execute without producing any warnings or error messages.

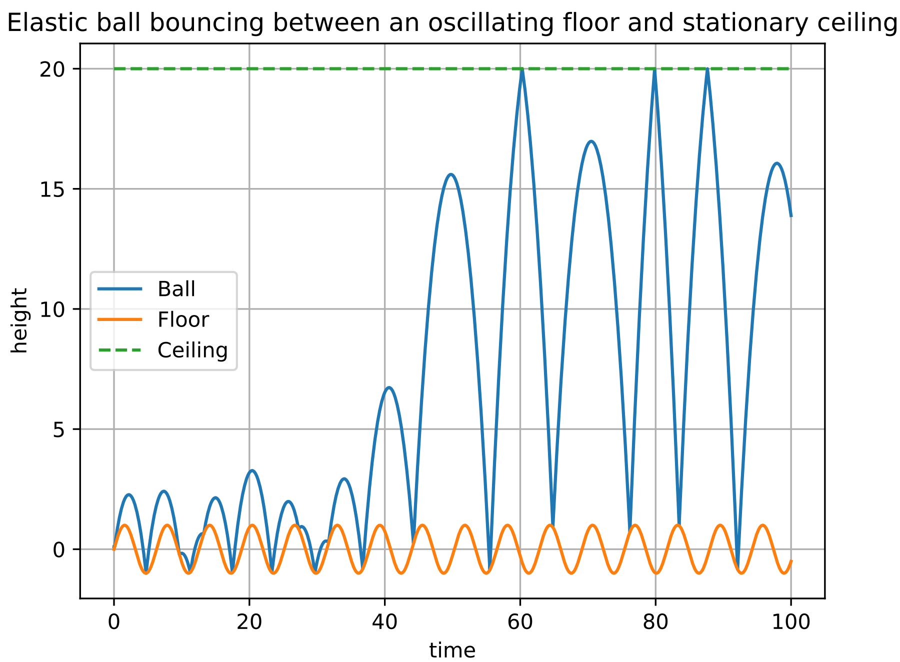

Figure 2: Expected plot for Problem 3(b) (color online).

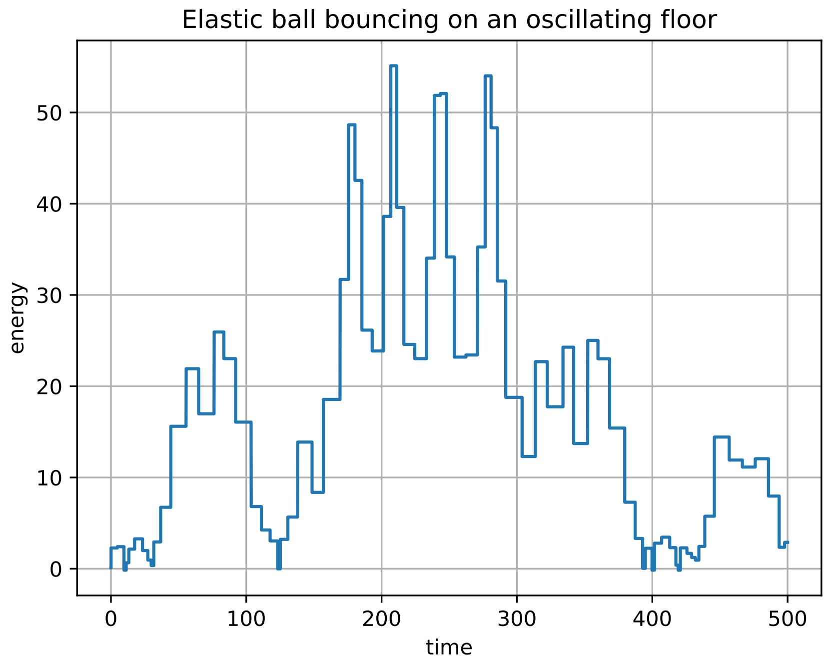

Figure 3: Expected plot for Problem 3(c).