Name: _____

Date: _____

Collaborators: _____

(If applicable, the collaborators submit their individually written assignments together)

| Question: | 1 | Total |
|---|---|---|
| Points: | 70 | 70 |
| Score: | | |

**Instructor/grader comments:**

1. Simpson's quadrature used for numerical evaluation of integrals, is as follows:

$$\int_a^b f(x)\,\mathrm{d}x \approx \frac{h}{3}\left(f(x_1)+4f(x_2)+2f(x_3)+4f(x_4)+\ldots+4f(x_{n-1})+f(x_n)\right)+O(h^p), \quad (1)$$

where

$$h=\frac{b-a}{n-1}, \quad (2)$$

$$x_i = a+h(i-1), \quad i=1,\ldots,n\,, \quad (3)$$

and $p$ is the order of accuracy of the quadrature. Note that the number of nodes, $n$, in Eq. (1) is an odd number (larger than 3).

(a) (5 points) Accept the assignment in GitHub Classroom, launch the codespace, open the template of the notebook for the assignment, `quadrature.ipynb`.

(b) (10 points) Use Markdown to fill in the blanks in the introductory part of the notebook.

(c) (20 points) Write a function `mysimpsons(fun, a, b, n)` that accepts the integrand (a julia function of a single argument), the integration limits, and the number of nodes, and returns the approximate numerical value of the integral.

Your function must check whether $n$ is indeed an odd number, $n>3$, and return the value `Inf` (as an indication of an error) if $n$ is an even number of $n=1,3$. You may use the functions `iseven()` and `isodd()`.

(d) (10 points) To test your function, evaluate the following elementary integrals using $n=11$.

$$\int_0^1 e^x\,\mathrm{d}x = e-1 \approx 1.72, \quad (4)$$

$$\int_1^2 x^2\,\mathrm{d}x = \frac{7}{3} \approx 2.33\,. \quad (5)$$

(e) (20 points) Consider the integral,

$$\int_0^\pi \sin(x)\,\mathrm{d}x = 2\,. \quad (6)$$

Evaluate this integral numerically for multiple values of $h$ using your function `mysimpsons(fun, a, b, n)`. Use $n=2^{i+1}+1$ for $i=1,\ldots,10$. Plot the absolute errors of the result, $\Delta(h)$, in appropriate axis (linear, semilog, loglog, etc.), and

"experimentally" determine the order of accuracy, $p$, of the Simpson's quadrature. Provide the legend, grid, title, axes labels for your plot. Describe your reasoning and the result of your numerical experiment.

(f) (5 points) Clean the cells of your jupyter notebook, save and close the notebook. Delete unneeded notebooks if you created any (e.g. `Untitled.ipynb`). Commit all your changes to the project and push them to the assignment's GitHub repository.

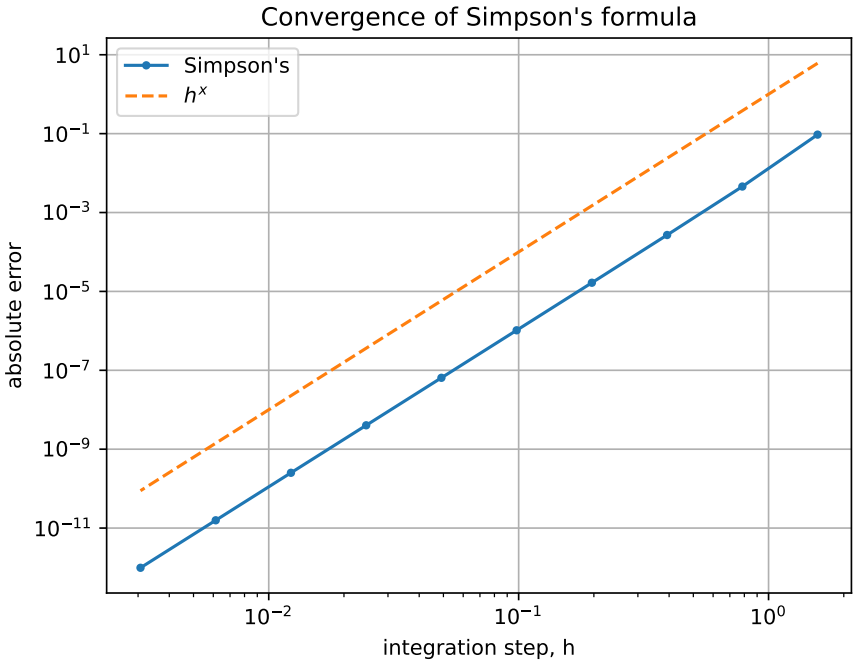


Figure 1: Expected graph in Problem 1. (colors online)