Name: ______________________________

Date: _______________________________

Collaborators: ____

(If applicable, the collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	5	30	30	5	70
Score:					

Instructor/grader comments:

1. (5 points) Accept the assignment in GitHub Classroom, launch the codespace. To work with the assignments, open the templates of the provided notebooks.

2. **Floating point numbers**

   Floating point numbers typically are represented in computers in the following binary form:

   $$x = \pm\left(1 + \frac{b_1}{2} + \frac{b_2}{2^2} + \ldots + \frac{b_d}{2^d}\right) \times 2^E,$$

   where $b_i \in \{0, 1\}$, $i = 1, \ldots, d$; E is an integer that can be positive, negative, or 0.

   (a) (10 points) What is the (approximate) value of machine epsilon for a microprocessor that uses $d = 10$? Explain in the space below.

   (b) (10 points) For the same microprocessor, how many floating point numbers x, such that $4 \leq x < 16$ are there? Explain in the space below.

(c) (10 points) For the same microprocessor, assuming that the smallest possible value of E is -15 and the largest possible value of E is 16, what are (approximately) the smallest and the largest positive floating point number? Explain in the space below. Present your answers as a power of 2 and as a nearest power of 10.

3. **Buckling and large deformations of rods**

Buckling instability is a sudden change of the shape of a straight rod that is compressed longitudinally. Buckling does not happen until the compressive forces on the road terminals exceed a certain threshold, called *buckling threshold*.

For reference, the buckling threshold of a slender rod, first determined by Euler, is as follows

$$F_B = \frac{\pi^2 EI}{L^2}.$$

Here L is the length of the rod, E is the Young's modulus of the rod material, and I is the area moment of inertia of the cross section of the rod.

For a wooden walking stick of length $L = 1$ m and circular cross section of diameter $D = 2$ cm, with wood Young's modulus $E = 10^{10}$ Pa, the buckling threshold is $F_B = 775$ N, corresponding to the weight of mass $m = 79$ kg.

This assignment deals with large deformation of a stringed bow which may be viewed as a straight rod that has been brought beyond the buckling threshold and is kept in mechanical equilibrium by the tension in the bowstring. In this case, the deflection of the rod from its non-deformed equilibrium is not small compared to the dimension of the bow, but the strains in the material are still small as long as the radius of curvature of the bow is much larger than the transverse dimensions of the rod. This permit us to use the linear elasticity theory to analyze large deflections of the bow.

Figure 1: The geometry of a stringed bow with opening angle α.

The results of the theoretical analysis of large deflection of the bow are as follows:

- The deformation of the bow can be described by a single parameter - the so called *opening angle*, α. See Fig. 1 for the sketch of a bow geometry and the definition of opening angle.
- It is natural to measure the tension force in the bow string in units of the buckling threshold,
$$f(\alpha) = \frac{F}{F_B}. \tag{1}$$
- The dimensionless tension in the string,
$$f(\alpha) = \frac{1}{\pi^2} I_1^2(\alpha) \tag{2}$$
where
$$I_1(\alpha) = \sqrt{2} \int_0^\alpha \frac{\mathrm{d}x}{\sqrt{\cos(x) - \cos(\alpha)}} \tag{3}$$

(a) (10 points) The integral in Eq. (3) is written in a form that is not suitable for its numerical evaluations: the term $\cos(x) - \cos(\alpha)$ would cause catastrophic cancellations in the denominators of the integrands when $x \to \alpha$.

One of the way to avoid the catastrophic cancellations, is to rewrite the integrand in a form that doesn't contain a subtraction of very close floating point values.

Rewrite integral I_1, Eq. (3) in a form free from catastrophic cancellations. Write your derivations in the space below.

Hints: first use the trigonometric identity

$$\cos(x) - \cos(\alpha) = 2\sin\left(\frac{\alpha + x}{2}\right)\sin\left(\frac{\alpha - x}{2}\right). \tag{4}$$

Next, introduce a new integration variable, y,

$$y = \alpha - x, \quad 0 \le y \le \alpha, \quad x = \alpha - y, \quad \mathrm{d}x = -\mathrm{d}y, \quad \alpha + x = 2\alpha - y. \tag{5}$$

(b) (10 points) Write a Julia function, `I1(alpha)`, that accept the value of the opening angle of the bow (in radians), and return the numerical value of the integral $I_1(\alpha)$. Use Julia package `QuadGK`.

(c) (10 points) Plot the graphs $f(\alpha)$ for $5^\circ \leq \alpha \leq 150^\circ$ (angle in degrees). Use at least 30 data points. For conversion from degrees to radians use the following relation:

$$\alpha_{\text{rad}} = \frac{\pi}{180}\alpha_{\text{deg}} \tag{6}$$

Provide axes labels, grid, title for your graph.

Use your graph to determine (approximately) the dimensionless tension in the bow string for the opening angle 100^{circ}. What is the value of string tension, in Newtons, for a bow made from the wooden stick that was discussed earlier?

See Fig. 2 for a possible graph your code is expected to produce.

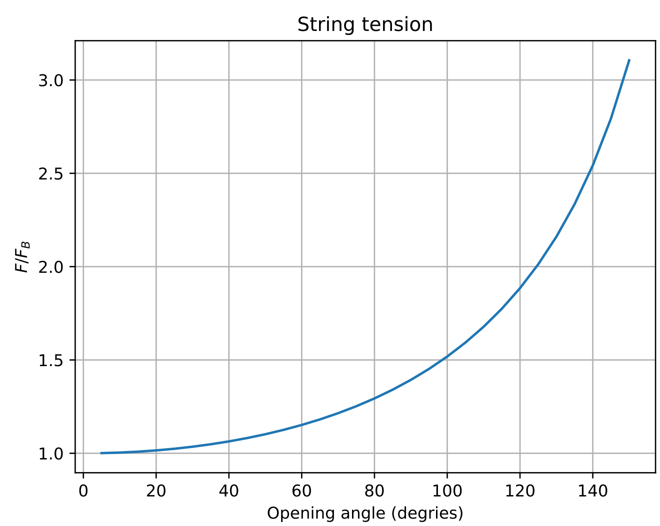

Figure 2: Expected graph for $f(\alpha)$ in Problem 3.

4. (5 points) **Submission of the assignment**

Clean the cells of your jupyter notebook(s). Delete unneeded notebooks if you created any (e.g. `Untitled.ipynb`). Commit all your changes to the project and push the updated files (including `Manifest.toml` and `Project.toml` if relevant) to the assignment's GitHub repository.