

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Collaborators: \_\_\_\_\_

(If applicable, the collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	5	25	30	5	65
Score:					

**Instructor/grader comments:**

1. (5 points) Accept the assignment in GitHub Classroom, launch the codespace. To work with the assignments, open the templates of the provided notebooks.

2. **White dwarfs: working with observational data**

White dwarfs are the final evolutionary state of stars whose mass is not high enough to become a neutron star or a black hole. After the hydrogen-fusing ( $H \rightarrow He$ ) lifetime of a star ends, such a star fuses helium to carbon and oxygen, ( $He \rightarrow C, O$ ). If a star has insufficient mass to generate the core temperatures required to further fuse carbon and oxygen, an inert mass of carbon and oxygen builds up at its center. After shedding its outer layers, the star leaves behind the core, which is the white dwarf.

Since the material in a white dwarf no longer undergoes fusion reactions, the star is not supported against gravitational collapse by the heat generated by fusion. It is supported only by a much weaker *electron degeneracy* pressure, causing the star to be very small in size and extremely dense.

Surprisingly, the larger is the mass of a white dwarf, the smaller is its radius. There is a characteristic mass, called *Chandrasekhar mass* or *Chandrasekhar limit*, above which electron degeneracy pressure in the star's core is insufficient to balance the star's own gravitational self-attraction. A star with a mass greater than the limit is evolving into a neutron star or black hole. Chandrasekhar limit corresponds to the point where the graph of radius of white dwarf vs mass,  $r(m)$  crosses the  $m$  axis.

The purpose of the assignment is to use astronomical observational data to estimate the Chandrasekhar limit.

- (a) (5 points) Use the provided in the notebook Julia code that downloads the catalog of white dwarfs observational data from The VizieR database of astronomical catalogues and clean the data.
- (b) (10 points) Use the function `linear_regression` that we discussed in class to find the best linear fit of the data. On the same figure plot the data and the fit. Provide axes labels, grid, legend, title.

The expected plot is shown in Fig. 1.

- (c) (10 points) Use the parameters of your fit to approximately determine (and print) the Chandrasekhar limit for white dwarfs. Discuss how you obtained the approximation and how close the result of using the linear fit is to the result of a better theory (search online).

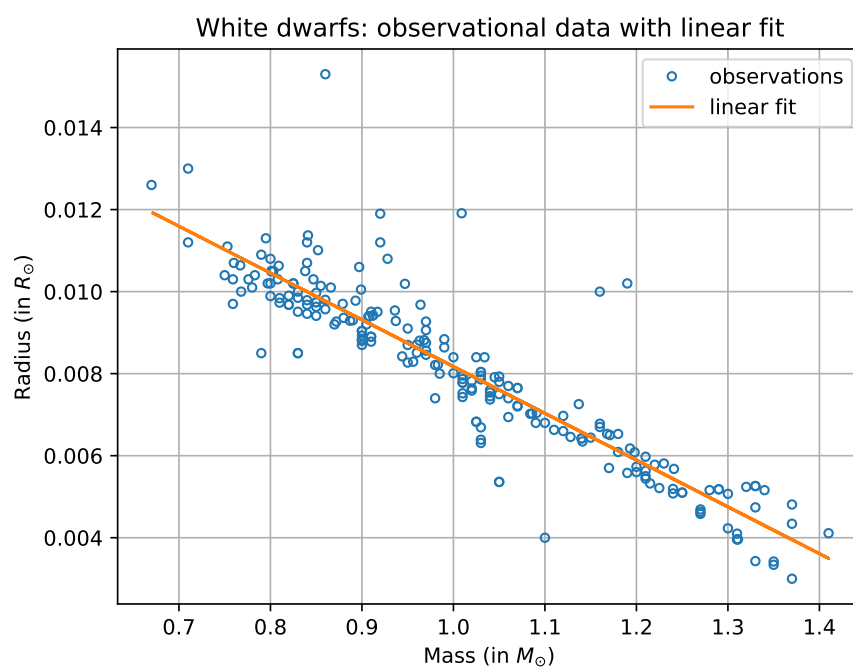


Figure 1: Expected plot for Problem 2(b).

### 3. Fermi acceleration

Fermi acceleration is a mechanism, first suggested by Enrico Fermi in 1949, to explain acceleration of particles in cosmic rays. Fermi studied charged particles being reflected by the moving interstellar magnetic field and either gaining or losing energy, depending on whether the "magnetic mirror" is approaching or receding. In a typical environment, Fermi argued, the probability of a head-on collision is greater than a head-tail collision, so particles would, on average, be accelerated. Since then Fermi acceleration has been used to explain a number of natural phenomena and several simple mathematical models demonstrating Fermi acceleration have been proposed.

One of a simple mathematical models proposed to describe the Fermi acceleration is the so-called The Fermi-Ulam model. Consider a particle that moves vertically in the uniform gravitational field with the acceleration of gravity  $g$ . The particle elastically collides with a harmonically oscillating floor and the stationary ceiling. The floor and the ceiling represent the magnetic mirrors which reflect the cosmic particles.

- (a) (10 points) Complete the code provide in the assignment.
- (b) (10 points) Follow the motion of the ball for  $t_{\text{span}} = (0.0, 100.0)$ . Plot the trajectory of the ball as well as the position of the floor. Provide the axes labels, grids, title, legend

The expected plot is shown in Fig. 2.

- (c) (10 points) Follow the motion of the particle for  $t_{\text{span}} = (0.0, 1000.0)$ . Plot energy of the ball as a function of time.. Provide the axes labels, grid, title.

The expected plot is shown in Fig. 3.

Does the particle monotonously gains energy? Could the Fermi-Ulam model be used to explain observed acceleration of charged particles in cosmic rays? Present your observations and the conclusion.

### 4. (5 points) Submission of the assignment.

Clean the cells of your jupyter notebook(s), save and close the notebook(s). Delete unneeded notebooks if you created any (e.g. Untitled.ipynb). Commit all your changes to the project and push them to the assignment's GitHub repository.

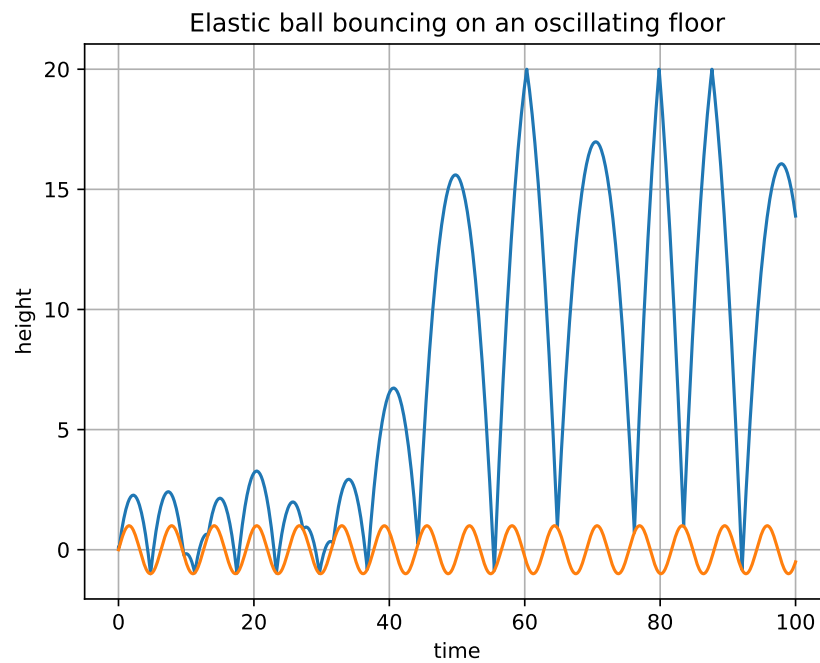


Figure 2: Expected plot for Problem 3(b) (color online).

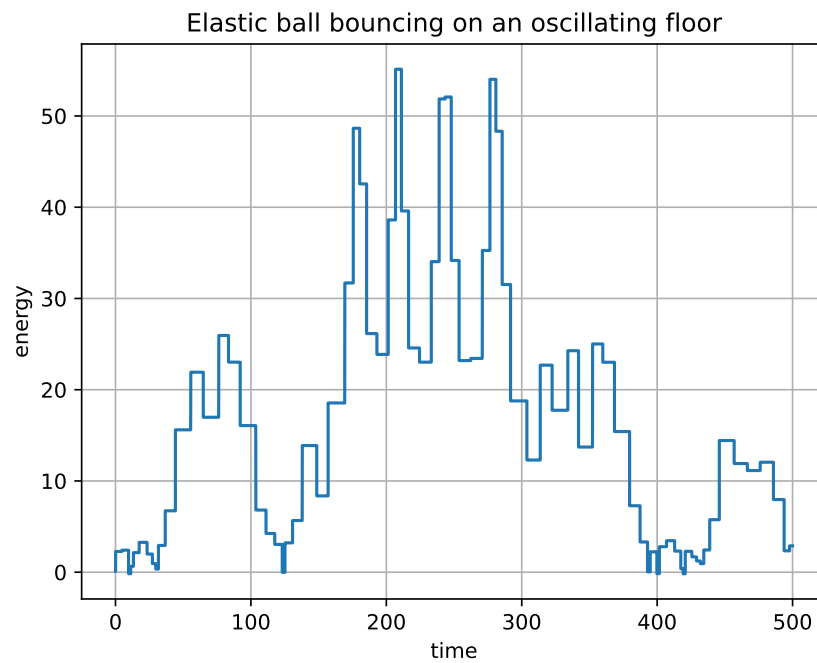


Figure 3: Expected plot for Problem 3(c).