MEAN-FIELD THEORY OF FERROMAGNETISM

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https://www.phys.uconn.edu/~rozman/Courses/P2200_23F/

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Mean-field approximation

Consider system of N spins s_i, $i = 1, \ldots, N$, $s_i = \pm 1$ on a periodic lattice. Each spin interacts with its close neighbors on the lattice. The energy of the system is as follows:

$$E_N = -J \sum_{i=1}^{N} s_i \sum_{\langle ij \rangle, i \neq j} s_j. \tag{1}$$

Here J, $J > 0$, is the coupling constant, such that the energy is minimized when the neighboring spins point in the same direction.

We assume that each spin interacts on average in the same way with its neighbors. This means that we can replace spin by their average value plus small fluctuations around the mean field.

$$s_i \longrightarrow \langle s \rangle + \delta_i, \quad \langle s \rangle \equiv \frac{1}{N} \sum_i s_i. \tag{2}$$

Let's ignore the effect of fluctuations, i.e. $\delta_i \to 0$. Then, the energy of the system can be rewritten as follows:

$$E_N = -\sum_{i=1}^{N} s_i \left(J \sum_{\langle ij \rangle} \langle s \rangle \right) = -B_{\text{eff}} \sum_{i=1}^{N} s_i, \tag{3}$$

where

$$B_{\text{eff}} = J\, z \langle s \rangle, \tag{4}$$

and z is the number of nearest neighbours.

The expression Eq. (3) is the energy of *non-interacting spins* in the effective field B_{eff}.

For non-interacting spins in the field B_{eff}, the average value of the spin, $\langle s \rangle$, is determined as follows:

$$\langle s \rangle = \tanh\left(\frac{B_{\text{eff}}}{k_B T}\right) = \tanh\left(\frac{Jz}{k_B T}\langle s \rangle\right), \tag{5}$$

where T is the temperature of the system, k_B is the Boltzmann constant.

Introducing the notation T_c for the so called *critical temperature*,

$$T_c \equiv \frac{Jz}{k_B}, \tag{6}$$

we can write Eq. (5) in the following universal form:

$$\langle s \rangle = \tanh\left(\frac{T_c}{T}\langle s \rangle\right). \tag{7}$$

Numerical solution

```
using PyPlot
using Roots

# tau = T/T_c
magnetization(m, tau) = m - tanh(m/tau)

taumin = 0.2
taumax = 1.0
np = 201

tau = range(taumin, taumax, np)
mag = zeros(np);

for i = 1:np-1
    mag[i] = find_zero(magnetization, (0.001, 1.0), tau[i])
end

plot(tau, mag)
grid(true)
xlabel(L"T/T_c")
ylabel(L"\langle s \rangle")
title("Spontaneous magnetization in ferromagnet")
```