Name: ______________________________

Date: ______________________________

Collaborators: ____

(If applicable, the collaborators submit their individually written assignments together)

Question:	1	2	3	Total
Points:	40	25	15	80
Score:				

Instructor/grader comments:

Use jupyter notebook interface to write the code for this homework assignment. Place all code in a single notebook file (call it e.g. **hw04.ipynb**).

1. Consider the Initial Value Problem (IVP) for a first order differential equation:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y), \quad a \le t \le b, \quad y(a) = \alpha. \tag{1}$$

A Runge-Kutta method for solving Eq. (1), known as midpoint method, is the following algorithm:

$$\begin{aligned} k_1 &= hf(t_i, y_i), \\ k_2 &= hf(t_i + \frac{h}{2}, y_i + \frac{k_1}{2}), \\ y_{i+1} &= y_i + k_2, \end{aligned} \tag{2}$$

where $i = 1, \ldots, n-1$, $t_i = a + (i-1)h$, h is the integration step, $h = (b-a)/(n-1)$.

(a) (10 points) On your virtual machine, **once per every project**: create a directory for your project (say `mkdir hw04`) and change to that directory; create an empty README.md file (say `echo " " > README.md`); download a sample .gitignore file and properly rename it (or copy it from one of your earlier projects). Start julia, activate the project and add packages you will use.

(b) (10 points) write a function `myrkmid(fun, a, b, n, y0)` that accepts as the arguments the function of two variables, `fun(t, y)` (the right-hand side of Eq. (1)), the integration limits `a` and `b`, the number of nodes `n`, and the initial value $y(a) = y0$, uses the Runge-Kutta midpoint method, and returns two vectors, t and y, where $t_i = a + (i-1)h$, $y_i = y(t_i)$, $i = 1, \ldots, n$ is the solution of the IVP at $t = t_i$.

(c) (10 points) Consider the IVP,

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y, \quad 0 \le t \le 5, \quad y(0) = 1, \tag{3}$$

with the exact solution

$$y_{\mathrm{ex}}(t) = e^t. \tag{4}$$

Solve IVP Eq. (3) using your function `myrkmid` for $n = 16$. On the same figure plot your numerical solutions and the exact solution Eq. (4). Provide the legend, grid, title, axes labels for your graph.

(d) (10 points) Solve IVP Eq. (3) using your function `myrkmid` for $n = 8, 16, \ldots, 2048, 4096$, i.e. $n(l) = 2^{l+3}$, $l = 1, \ldots, 9$.

Find the global errors of your solutions defined as the errors of solutions at the right hand end of the integration range:

$$\Delta(h_{n(l)}) = \left| y_{n(l)} - y_{\text{ex}}(b) \right|. \tag{5}$$

Plot Δ vs. the integration step h. By visual inspection determine the order of accuracy of the midpoint Runge-Kutta method. ((Use the appropriate style of plot axes.) Provide the legend, grid, title, axes labels for your graph. Describe your reasoning and the result of your numerical experiment in the README.md file of your git project.

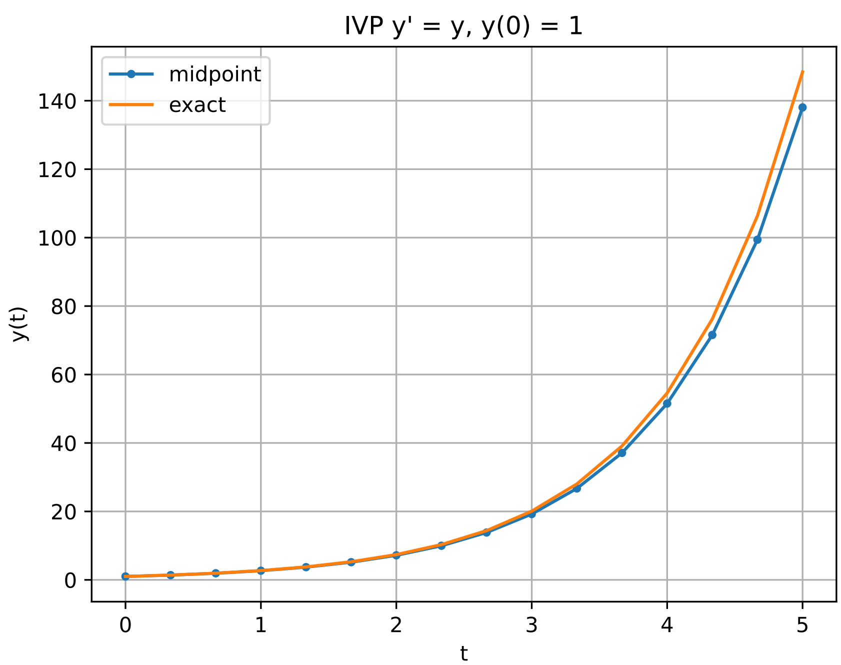

Figure 1: Expected graph in Problem 1 (c).

2. (25 points)

    1. On the GitLab: Create an empty GitLab project called **hw04** (name it exactly as shown).

    2. On the VM: Clean the cells of your jupyter notebook and save the notebook. Delete unneeded notebooks if you created ones (e.g. `Untitled.ipynb`). Initialize a git repository for your project. Check in your notebook, Project.toml and Manifest.toml, an empty README.md file, and your .gitignore file into the

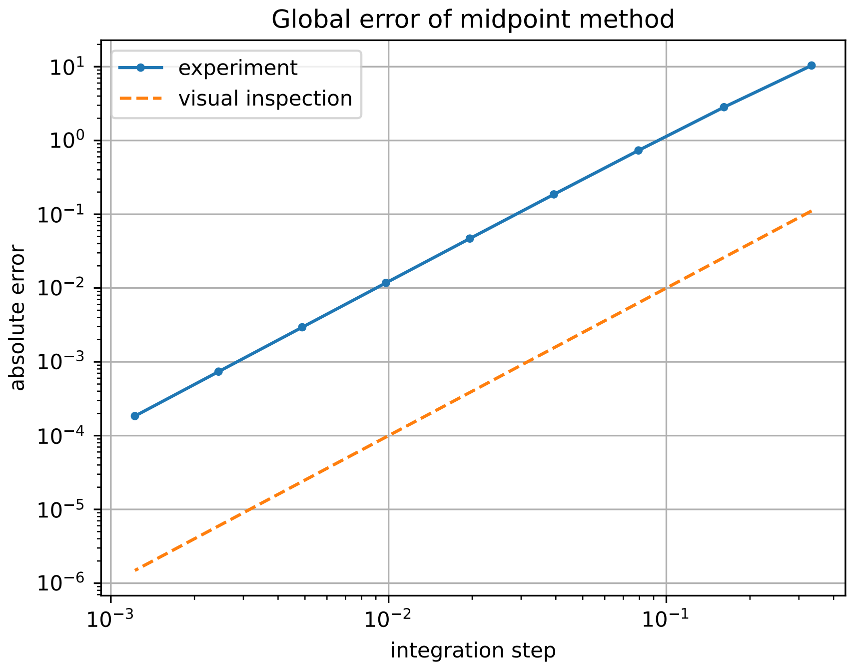

Figure 2: Expected graph in Problem 1 (d).

repository. Provide a meaningful commit message. Push the content of your git repository to GitLab hw04 project.

3. On the GitLab: Edit README.md file as requested in Problem 1.
4. On the VM: Pull the README.md file to your local git repository (`git pull`).
5. On the GitLab: Share the project with the instructor (GitLab user name p2200_23f_in) and grant him **Reporter** privileges.

I have synchronized the contents of my local and remote repositories that I created for hw04 assignment

Sign and date here: ______________________________

3. (15 points) In numerical analysis Gauss–Laguerre quadrature (named after Carl Gauss and Edmond Laguerre) is a method for approximating the value of integrals of the following kind:
$$\int_0^\infty e^{-x} f(x)\,\mathrm{d}x \approx \sum_{i=1}^{n} w_i f(x_i). \tag{6}$$
Determine the weight, w_1, and the coordinate, x_1, of one-point Gauss–Laguerre rule.
$$\int_0^\infty e^{-x} f(x)\,\mathrm{d}x \approx w_1 f(x_1). \tag{7}$$
Test your result by evaluating the following integral:
$$\int_0^\infty e^{-x} \sin\left(\frac{x}{\pi^2}\right) \mathrm{d}x \approx 0.1003. \tag{8}$$
Use julia as a calculator to evaluate the right hand side of Eq. (7).

Show all your work below.

For reference,
$$\int_0^\infty e^{-x}\,\mathrm{d}x = 1, \tag{9}$$
$$\int_0^\infty x e^{-x}\,\mathrm{d}x = 1. \tag{10}$$