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A simple determination of Hubble's constant

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Abstract

The aim of this paper is to make a determination of Hubble's constant from the experimental data on the magnitude and redshift of supernovae. We proposed a very simple approach that could also be very useful from a didactic point of view.

Keywords: Hubble constant, supernova, expansion of the universe

1. Introduction

In the 1920s, Edwin P Hubble, also thanks to the work of Slipher and Humason [1–3], discovered a relationship that is now known as Hubble's law. It states that the recessional velocity of a galaxy is proportional to its distance from us. Therefore $V = H_0 d$, where V is the galaxy's velocity, d is our distance from the galaxy and H_0 is the proportionality constant, called Hubble's constant. The physical meaning of Hubble's law was understood in the framework of Einstein general relativity and it was soon clear that this equation has a simple interpretation (appendix A) only for nearby galaxies and that H_0 was not really constant. In fact the subscripted '0' refers to its 'today' value, because, in general, H changes with cosmic time, being related to the scale factor that rules the expansion of the universe. Anyway, its value today, H_0, is one of the most important numbers in cosmology, because it may be used to calculate the size and the age of the Universe, and continues to be called Hubble's constant. As there has always been considerable uncertainty about its correct value, it is usually expressed by the scientific community in the form $H_0 = 100h_0\ \mathrm{km\,s^{-1}Mpc^{-1}}$ with $0.5 < h_0 < 1$. We assume this fact as the starting point of our analysis, therefore we want to determine Hubble's constant, searching for it only in this range of possible values.

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One of the methods used to estimate the value of Hubble's constant is based on the observation of a sample of variable stars called Cepheids. These kinds of stars are very important, because their periods of variability have been discovered to be related to their absolute luminosity [4–7]. They are called Cepheids because the first star of this type to be discovered was Delta Cephei in the constellation of Cepheus. The relationship between a Cepheid's luminosity and pulsation period is used to establish the galactic and extragalactic distance scales. Practically, it is possible to derive the absolute magnitude M of a Cepheid from the pulsation period and, by comparison with its apparent magnitude m, to determine the distance d in parsecs (pc) with the well known relation $M = m + 5 - 5\log(d/\text{pc})$. Let us remember that the magnitude is a measure of the amount of light that reaches us from a celestial body. Apparent magnitude m is a number that tells us how bright that star appears at its real distance from Earth, while the absolute magnitude M is the magnitude that the star would have if it were placed at a distance d of 10 parsecs from the Earth (appendix B).

Observing a great number of Cepheid variables and supernovae in galaxies at various distances, some astronomers have concluded that the value of Hubble's constant is equal to $73.8 \pm 2.4\,\text{km s}^{-1}\text{Mpc}^{-1}$ [8] others that it is $74.3 \pm 1.5\,(\text{stat.}) \pm 2.1\,(\text{syst.})\,\text{km s}^{-1}\text{Mpc}^{-1}$ [9]. There are also other estimations of Hubble's constant obtained with different methods (see figure 16 in [10]), but the last very precise value was estimated by the data of the Planck satellite [10]. The main goal of the Planck satellite was to draw the map of the anisotropy of the Cosmic Background Radiation. From the power spectrum of the radiation it is possible to derive a set of basic information about the Universe, including Hubble's constant. The so called ΛCDM cosmological model depends on six parameters which are determined by a fit of the experimental data on the Cosmic Background Radiation. This fit provides the current speed of expansion of the Universe, and therefore Hubble's constant with a value of $67.80 \pm 0.77\,\text{km s}^{-1}\text{Mpc}^{-1}$. It is evident that this determination of Hubble's constant is not equal to the previous ones inside the experimental errors. This is a motivation to try a new and different determination of the constant that must be very simple and low cost.

2. Magnitude–redshift relation

In order to estimate Hubble's constant, we will study the cosmological magnitude–redshift relation (Hubble's diagram), considering objects outside our Galaxy, so we will use a modified relationship between m of a standard candle and M, with the distance expressed in Megaparsec (appendix B):

$$m = M + 5\log\left(\frac{d_L}{\text{Mpc}}\right) + 25 \tag{1}$$

where the luminosity distance d_L is defined in terms of the ratio between L, the total energy emitted by a source in unit time at the epoch t_1 and the coordinate r_1, and F, the flux received at the epoch t_0 by an observer placed in r_0, so $d_L = \sqrt{L/4\pi F}$. In a rigorous approach the luminosity distance depends on the spatial curvature k, on the existence of a vacuum dark energy related to a cosmological constant Λ, on Hubble's constant and on the redshift z defined as in appendix A. The most general expression of d_L is complicated [11] and can be fully explained only starting from Einstein field equations of general relativity. This approach is beyond the scope of our paper but can be easily recovered in normal textbooks of cosmology [11, 12] and in simple reviews on this subject such as [13].

For example, for the Einstein–de Sitter flat universe [14, 15] ($k = 0$, $\Lambda = 0$), the luminosity distance is expressed in terms of redshift z by the formula

$$d_L = \frac{2c}{H_0}[(1 + z) - \sqrt{1 + z}]. \tag{2}$$

By introducing

$$\tilde{M} = M + 5\log\left(\frac{c}{H_0 Mpc}\right) + 25 \tag{3}$$

equation (1) becomes

$$m(z) = \tilde{M} + 5\log[2(1 + z - \sqrt{1 + z})]. \tag{4}$$

We can perform a first approximation of equation (4) considering $z \ll 1$ and obtaining:

$$m(z) = \tilde{M} + 5\log(z) \tag{5}$$

because $\sqrt{1 + z} \approx 1 + \frac{z}{2}$ and

$$D_L = 2[1 + z - \sqrt{1 + z}] \approx z \tag{6}$$

where we have defined the dimensionless 'Hubble free' luminosity distance as $D_L = H_0 d_L / c$.

So, there is a linear relationship linking the magnitude and the logarithm of the redshift called Hubble's diagram. In our paper we will consider only the objects with $z \ll 1$, so the corresponding Hubble diagram will be independent on the spatial curvature k and on the existence of a cosmological constant Λ, because the result of equation (6), $D_L = z$, holds in this approximation, not only for the Einstein–de Sitter universe, but also for models with nonvanishing k and Λ. The case $z \ll 1$, from a didactic point of view, can be easily explained also recurring to the Doppler effect (appendix A). On the other hand, our simplified model cannot be used to discriminate among the three possible kinds of spatial curvature (sphere, flat plane, saddle) or to determine the existence of a cosmological constant.

Our aim is to simplify further equation (5) trying to eliminate the logarithm with a series expansion. Since $c \cong 3 \cdot 10^5\,\mathrm{km\,s^{-1}}$ and imposing $H_0 = 3000 \cdot l_0$ km $\mathrm{s^{-1}Mpc^{-1}}$, where l_0 is a dimensionless constant, relation (5) becomes

$$m = M + 25 + 5\log\left(\frac{c}{3000\mathrm{Mpc}}\right) + 5\log\left(\frac{z}{l_0}\right) \tag{7}$$

$$m = M + 35 + 5\log\left(\frac{z}{l_0}\right). \tag{8}$$

As written in the introduction, we search for a value of H_0 between 50 and 100 $\mathrm{km\,s^{-1}Mpc^{-1}}$, so the constant l_0 must be within a range that goes from 0.0166 to 0.0333, and therefore to have a series expansion near to $z/l_0 = 1$, we must choose for our experimental determination only objects with redshift $0.0166 < z < 0.0333$.

We perform now our second approximation expanding in Taylor series near $z/l_0 = 1$ equation (8) and we get

$$m = M + 35 + 5(0.434)\left(\frac{z}{l_0} - 1\right). \tag{9}$$

In this way we have been able to reduce equation (4) to a very simple linear relation between magnitude and redshift

$$m = \alpha + \beta z \tag{10}$$

where

$$\begin{cases} \alpha = M + 35 - 5(0.434) \\ \quad \beta = \frac{5(0.434)}{l_0} = \frac{2.171}{l_0} \end{cases}. \tag{11}$$

The fit of the experimental data, achieved through the use of suitable software, allows us to obtain the values of α and β. Then, thanks to equations (11), we can calculate the absolute magnitude and Hubble's constant:

$$\begin{cases} \quad M = \alpha - 32.829 \\ H_0 = l_0 \cdot 3000 = \frac{3000 \cdot 2.171}{\beta} \end{cases}. \tag{12}$$

3. The experimental data

Thanks to satellites and large ground-based telescopes, in recent years it has been possible to study a huge number of supernovae. The data have been published in some catalogues that are available in specialized journals. To determine an experimental value of Hubble's constant, we used one of these catalogues and in particular the one collected by the 'Union 2 Compilation' [16] of the research team called 'Supernova Cosmology Project'. Of these experimental data we will take into account the supernovae with redshifts between 0.0166 and 0.0333 because only in this range of redshifts can we expand in series relationship (8). Therefore, among the 719 objects listed in the database, we have chosen the 90 supernovae in that range of redshift and then we have decided to discard only one, precisely the 2006 br, because compared to all the others it is characterized by too high a magnitude, which is probably due to a measurement error. Before performing the data processing using the remaining 89 objects, we introduce another parameter, the distance modulus μ

$$\mu = m - M. \tag{13}$$

In the catalogue (see the table below) the values of μ and $\Delta\mu$ are listed. The distance modulus of each supernova is corrected using a suitable stretch factor x_1, color factor c and fitted magnitude $M_0 = -18.598$ following the formula

$$\mu = m^{\mathrm{eff}} - M_0 \tag{14}$$

where

$$m^{\mathrm{eff}} = m + \gamma x_1 - \delta c \tag{15}$$

and the parameters used by [16] are $\gamma = 0.12$ and $\delta = 2.51$. The values of m are corrected in this way by x_1 and c in order to make the sample of supernovae more homogeneous. Actually they, used as standard candles in Hubble's diagram, must have about the same intrinsic luminosity. So we can choose to fit the data using either the listed values of m or the corrected values of distance modulus μ. In the second case we expect a reduction in the errors and an increase of the correlation coefficient of the fitted relation. So equation (10) is substituted by

$$\mu = \tilde{\alpha} + \beta z \tag{16}$$

where the corrected parameters are

$$\begin{cases} \tilde{\alpha} = M - M_0 + 35 - 5(0.434) \\ \beta = \frac{5(0.434)}{l_0} = \frac{2.171}{l_0} \end{cases}. \tag{17}$$

Hence

$$\begin{cases} M = \tilde{\alpha} + M_0 - 32.829 \\ H_0 = l_0 \cdot 3000 = \frac{3000 \cdot 2.171}{\beta} \end{cases}. \tag{18}$$

Fitting the experimental data with the routine in 'Mathematica' minimizing the χ^2 taking into account also the errors $\Delta\mu$, we obtain

$$\begin{array}{llll} \tilde{\alpha} & \Delta\tilde{\alpha} & \beta & \Delta\beta \\ 31.931 & 0.095 & 99.714 & 3.679 \end{array} \tag{19}$$

and from equations (18) and considering an error in Hubble's constant $\Delta H_0 = \frac{3000 \cdot 2.171}{\beta^2} \Delta\beta$ we find

$$M = -19.50 \pm 0.09 \tag{20}$$

and

$$H_0 = 65.3 \pm 2.4 \text{ km s}^{-1}\text{ Mpc}^{-1} \tag{21}$$

with a correlation coefficient $r = 0.95$ and a reduced $\chi^2 = 0.98$. Our result has the same error as the value determined using Cepheids [8] but it is compatible inside the errors with the Planck satellite measurement.

A posteriori we can also verify that the approximation $z/l_0 \simeq 1$ works fairly well because, from (11) and (19), we have $l_0 = 0.02177$ and hence $0.762 < z/l_0 < 1.529$. The approximation might work better if we reduce the interval of accepted redshifts, but in this way we would have a smaller sample for the statistical analysis.

SN	z	m	Δm	x_1	Δx_1	c	Δc	μ	$\Delta\mu$
1993*ah*	0.0285	16.86	0.19	−2.26	0.93	0.23	0.09	34.61	0.23
1992*bo*	0.0172	15.75	0.13	−2.68	0.18	0.03	0.02	33.93	0.20
1992*bc*	0.0196	15.07	0.11	0.51	0.10	−0.05	0.01	33.86	0.18
1992*ag*	0.0273	16.26	0.09	−0.13	0.21	0.19	0.02	34.35	0.18
1992*p*	0.0265	16.03	0.09	0.70	0.58	− 0.02	0.02	34.75	0.19
1990*o*	0.0306	16.19	0.08	0.39	0.29	− 0.00	0.03	34.82	0.17
2001*cz*	0.0163	15.02	0.14	0.04	0.14	0.12	0.01	33.31	0.14
2001*ba*	0.0305	16.19	0.07	0.00	0.14	− 0.04	0.01	34.87	0.09
2000*ca*	0.0245	15.53	0.09	0.35	0.20	− 0.07	0.01	34.33	0.10
2000*bh*	0.0240	15.92	0.10	−0.55	0.17	0.08	0.03	34.24	0.11
1999*gp*	0.0260	16.00	0.09	1.64	0.21	0.06	0.02	34.63	0.11
1994*m*	0.0243	16.27	0.09	−1.74	0.18	0.11	0.02	34.37	0.18
1995*ak*	0.0220	15.94	0.10	−1.53	0.20	0.09	0.03	34.12	0.19
1996*c*	0.0275	16.59	0.08	0.39	0.19	0.12	0.02	34.92	0.18
1996*bv*	0.0167	15.28	0.14	0.93	0.49	0.19	0.02	33.49	0.21
1996*bo*	0.0163	15.82	0.13	− 1.04	0.18	0.40	0.01	33.27	0.21
2000*fa*	0.0218	15.86	0.10	0.37	0.16	0.10	0.02	34.24	0.23
2000*dk*	0.0164	15.34	0.13	− 2.60	0.29	0.06	0.02	33.45	0.25
2000*cn*	0.0232	16.57	0.10	− 2.50	0.29	0.20	0.02	34.35	0.23
1999*gd*	0.0193	16.91	0.12	− 1.08	0.27	0.46	0.02	34.20	0.24
1999*ek*	0.0176	15.59	0.18	− 1.08	0.15	0.17	0.01	33.61	0.27
1999*cc*	0.0315	16.77	0.07	− 1.89	0.28	0.05	0.02	34.99	0.22
1998*eg*	0.0235	16.08	0.10	− 0.79	0.35	0.05	0.02	34.44	0.23
1998*ef*	0.0167	14.80	0.13	− 1.23	0.26	− 0.02	0.02	33.28	0.25
1998*co*	0.0170	15.65	0.14	− 2.33	1.54	0.12	0.05	33.65	0.31
1998*ab*	0.0279	16.11	0.08	0.04	0.14	0.14	0.02	34.35	0.22
1998*v*	0.0172	15.07	0..14	− 0.27	0.19	0.04	0.01	33.53	0.25
1997*dg*	0.0300	16.82	0.08	− 0.84	0.28	0.02	0.02	35.24	0.22
1997*y*	0.0166	15.31	0.13	− 1.22	0.21	0.06	0.02	33.61	0.25
2001*ay*	0.0309	16.71	0.08	3.06	0.37	0.19	0.03	35.19	0.18
2001*da*	0.0160	15.43	0.14	− 1.70	0.61	0.15	0.04	33.44	0.22
2001*g*	0.0173	14.91	0.14	− 0.08	0.36	− 0.02	0.05	33.52	0.21
2001*ie*	0.0312	16.56	0.09	− 1.26	0.23	0.04	0.03	34.89	0.17
2001*n*	0.0221	16.51	0.10	− 0.44	0.27	0.35	0.02	34.16	0.18
2001*v*	0.0160	14.54	0.14	0.77	0.14	0.05	0.02	33.09	0.20
2002*bf*	0.0249	16.51	0.11	− 2.30	0.46	0.30	0.04	34.06	0.19
2002*ck*	0.0303	16.24	0.09	− 0.04	0.16	− 0.03	0.02	34.90	0.17
2002*de*	0.0283	16.62	0.08	0.08	0.38	0.17	0.01	34.78	0.17
2002*he*	0.0248	16.23	0.10	− 2.06	0.26	0.01	0.02	34.53	0.18
2002*hu*	0.0292	16.59	0.08	0.27	0.17	− 0.02	0.01	35.26	0.17
2002*hw*	0.0163	16.66	0.14	− 2.30	0.27	0.50	0.03	33.70	0.21
2002*jy*	0.0187	15.69	0.12	0.77	0.19	0.02	0.01	34.31	0.19
2002*kf*	0.0195	15.62	0.12	− 1.35	0.19	0.00	0.02	34.03	0.19
2003*ch*	0.0256	16.66	0.09	− 1.62	0.30	0.04	0.02	34.95	0.18

SN	z	m	Δm	x_1	Δx_1	c	Δc	μ	$\Delta\mu$
2003*it*	0.0240	16.32	0.10	−1.82	0.37	0.10	0.03	34.44	0.19
2003*u*	0.0261	16.46	0.09	−2.60	0.57	0.04	0.04	34.63	0.19
2003*w*	0.0211	15.84	0.10	− 0.25	0.11	0.19	0.01	33.92	0.18
2004*as*	0.0321	16.96	0.07	0.01	0.22	0.15	0.02	35.16	0.17
2004*bg*	0.0221	15.46	0.10	0.35	0.18	− 0.04	0.02	34.19	0.18
2004*ef*	0.0298	16.86	0.08	− 2.05	0.47	0.17	0.04	34.78	0.21
2005*eq*.	0.0284	16.22	0.08	1.10	0.13	0.04	0.01	34.84	0.17
2005*iq*	0.0330	16.79	0.07	− 1.41	0.19	− 0.02	0.01	35.26	0.16
2005*ki*	0.0204	15.55	0.11	− 2.06	0.16	− 0.04	0.03	33.99	0.19
2005*ls*	0.0205	16.12	0.11	0.50	0.20	0.36	0.01	33.87	0.18
2005*mc*	0.0260	17.23	0.09	− 3.10	0.27	0.32	0.02	34.63	0.18
2005*ms*	0.0259	16.11	0.09	0.18	0.12	0.01	0.01	34.68	0.17
2005*na*	0.0268	15.99	0.08	− 0.75	0.14	− 0.04	0.01	34.58	0.17
2006*ac*	0.0239	16.14	0.09	− 1.08	0.12	0.12	0.01	34.30	0.17
2006*ar*	0.0229	16.44	0.10	− 0.76	0.30	0.19	0.01	34.46	0.18
2006*ax*	0.0180	15.01	0.12	0.01	0.07	− 0.03	0.01	33.67	0.19
2006*az*	0.0315	16.44	0.07	− 1.65	0.09	− 0.04	0.01	34.92	0.16
2006*bq*	0.0215	16.13	0.10	− 1.63	0.13	0.13	0.01	34.19	0.18
2006*br*	0.0255	18.89	0.09	− 1.59	0.57	0.93	0.02	34.95	0.19
2006*bt*	0.0325	16.90	0.07	0.16	0.10	017	0.01	35.07	0.16
2006*bw*	0.0308	17.41	0.08	− 2.14	0.27	0.34	0.03	34.89	0.17
2006*cc*	0.0327	17.75	0.07	0.19	0.09	0.40	0.01	35.35	0.16
2006*ej*	0.0192	15.74	0.12	− 1.56	0.16	0.05	0.02	34.00	0.19
2006*en*	0.0308	16.74	0.08	− 0.69	0.14	0.08	0.02	35.05	0.17
2006*et*	0.0212	15.92	0.11	0.75	0.26	0.20	0.02	34.11	0.18
2006*gj*	0.0277	17.63	0.09	− 2.09	0.28	0.40	0.02	34.96	0.18
2006*kf*	0.0208	15.80	0.12	− 2.85	0.23	− 0.01	0.03	34.07	0.20
2006*le*	0.0173	14.73	0.16	0.63	0.08	− 0.04	0.01	33.50	0.22
2006*mp*	0.0233	15.95	0.09	0.77	0.23	0.06	0.01	34.46	0.18
2006*os*	0.0321	17.57	0.08	− 0.72	0.16	0.47	0.02	34.90	0.17
2006*qo*	0.0308	16.78	0.07	0.31	0.10	0.22	0.01	34.85	0.16
2006*s*	0.0329	16.81	0.07	0.65	0.12	0.11	0.01	35.20	0.16
2006*sr*	0.0230	16.11	0.10	− 1.51	0.23	0.07	0.01	34.34	0.18
2006*te*	0.0321	16.50	0.08	− 0.36	0.18	− 0.04	0.02	35.14	0.17
2007*ai*	0.0320	16.85	0.10	1.28	0.26	0.20	0.02	35.09	0.19
2007*au*	0.0209	16.58	0.11	− 3.50	0.26	0.31	0.03	33.96	0.19
2007*bc*	0.0219	15.86	0.10	− 1.69	0.18	0.05	0.02	34.11	0.18
2007*bd*	0.0320	16.55	0.07	− 1.78	0.16	0.03	0.02	34.85	0.17
2007*ci*	0.0192	15.89	0.12	− 3.20	0.23	0.13	0.03	33.75	0.19
2007*co*	0.0266	16.43	0.09	− 0.39	0.09	0.15	0.01	34.59	0.17
2007*cq*	0.0247	15.79	0.09	− 0.62	0.16	0.05	0.01	34.18	0.17
2007*f*	0.0242	15.86	0.09	0.42	0.08	0.01	0.01	34.47	0.17
2007*qe*	0.0229	16.02	0.10	0.53	0.08	0.10	0.01	34.40	0.18
2007*r*	0.0312	16.60	0.07	− 1.76	0.16	− 0.07	0.02	35.15	0.17
2008*bf*	0.0251	15.68	0.09	0.25	0.12	0.03	0.01	34.21	0.17
2008*l*	0.0189	15.11	0.13	− 1.88	0.28	− 0.07	0.04	33.65	0.20

4. Conclusion

We have determined the value of Hubble's constant using data of magnitude and redshift of nearby supernovae. Our error is certainly greater than the value found by the Planck satellite, but we have obtained a highly reliable result. Furthermore, our estimation is closer to the Planck Collaboration's value than the one obtained using Cepheids. This exercise can also be very useful from the didactic point of view. Using basic notions of mathematics, a catalogue of data available free from the web, and suitable software to find the best fit of a linear relation, even high school students can find the value of a very important cosmological constant. In conclusion, we have shown that determining a constant that is so important for the evolution of the entire Universe has today become as easy as a simple homework, and this may be exciting for both scientists and students.

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Appendix A

A didactic approach, useful for high school students, consists in explaining Hubble's law in the light of the Doppler effect [17]. In special relativity, given a source of electromagnetic waves at rest in an inertial system S', moving with a constant velocity v with respect to another system S, the wavelength of the radiation emitted in S', denoted by λ_{em}, is related to the wavelength λ_{ob}, received by the observer in the system S, by the formula

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{22}$$

where the red-shift z is defined as

$$z = \frac{\lambda_{\mathrm{ob}} - \lambda_{\mathrm{em}}}{\lambda_{\mathrm{em}}}. \tag{23}$$

From (22), in the nonrelativistic limit, $z \approx v/c$. Actually, if one measures a red-shift proportional to the distance, one can write $z \approx H_0 d/c$ and interpret it as a recessional velocity v, finding Hubble's law in the form $v = H_0 d$. But this 'didactic' simplified interpretation hides some problems because the origin of the cosmological red-shift is different from the special relativistic Doppler shift [18]. Furthermore, the simple relation

$$d = \frac{cz}{H_0}, \tag{24}$$

obtained with this interpretation, is not always right. It works only for nearby galaxies ($z \ll 1$), otherwise the distance has a more complicated dependence on red-shift and on other cosmological parameters.

Anyway, for $z \ll 1$, introducing the new variable $D \equiv H_0 d/c$, we have $D \approx z$, which is result (6) written in the text.

Appendix B

For historical reasons the apparent magnitude m of a star in the photometric x- band (x could be U, B, V, etc.) is defined as

$$m_x = -2.5 \log\left(\frac{F_x}{C}\right) \tag{25}$$

where C is a constant depending on the x-band and F is the flux received by the observer on Earth related to the absolute luminosity L of the star and to its distance d by the formula

$$F = \frac{L}{4\pi d^2}. \tag{26}$$

In this way the difference in magnitude between two stars, in the same band x, is:

$$m_1 - m_2 = -2.5 \log\left(\frac{F_1}{F_2}\right) = -2.5 \log\left(\frac{4\pi d_2^2 L_1}{4\pi d_1^2 L_2}\right). \tag{27}$$

If we define the absolute magnitude M of a star as its apparent magnitude at a conventional distance $d = 10$ parsecs from the Earth, we obtain the relation between apparent and absolute magnitude of the same star ($L_1 = L_2$ and $d_1 = 10$ pc)

$$M - m = -2.5 \log\left(\frac{d^2}{10^2}\right) = -5 \log\left(\frac{d}{\mathrm{pc}}\right) + 5. \tag{28}$$

If the distance is measured in Megaparsec (1 Mpc $= 10^6$ pc), we have:

$$M - m = -5 \log\left(\frac{d \cdot 10^6}{\mathrm{Mpc}}\right) + 5 = -5 \log\left(\frac{d}{\mathrm{Mpc}}\right) - 25 \tag{29}$$

that is equation (1) in the text. Substituting in it equation (24), Hubble's constant emerges in the magnitude–redshift relation that holds for $z \ll 1$.

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