Name: ______________________________

Date: ______________________________

Collaborators: ____

(If applicable, the collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	5	40	15	15	75
Score:					

Instructor/grader comments:

Use jupyter notebook interface to write the code for this homework assignment. Create a directory for the assignment (say **hw05**); change to that directory and work in there. When you start julia inside the folder (for the first time only), activate the project and add packages that you will use (IJulia, PyPlot, OrdinaryDiffEq, etc.).

1. (5 points) I watched the video Spontaneous Synchronization which is a part of Homework 5 assignment.

   Sign and date here: ______________________________

2. The second order non-linear autonomous differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \varepsilon\left(x^2 - 1\right)\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0, \quad \varepsilon > 0 \tag{1}$$

   is called van der Pol equation. The parameter ε is positive and indicates the nonlinearity and the strength of the damping. The equation models a non-conservative system in which energy is added to and subtracted from. The sign of the "coefficient" in the damping term in Eq. (1), $\left(x^2 - 1\right)$ changes, depending whether $|x|$ is larger or smaller than one, describing the inflow and outflow of the energy.

   The equation was originally proposed in the 1920s to describe stable oscillations in electrical circuits employing vacuum tubes.

   Van der Pol oscillator is the example of a system that exibits the so called *limit cycle*. A limit cycle is an isolated closed trajectory $\dot{x}(x)$ in the phase space. Isolated means that neighboring trajectories are not closed; they spiral either toward or away from the limit cycle. If all neighboring trajectories approach the limit cycle, we say the limit cycle is stable or attracting. Otherwise the limit cycle is in general unstable.

   Stable limit cycles model systems, e.g. the beating of a heart, that exhibit self-sustained oscillations. These systems oscillate even in the absence of external periodic forcing. There is a standard oscillation of some preferred period, waveform, and amplitude. If the system is perturbed slightly, it returns to the standard cycle.

   Limit cycles are inherently nonlinear phenomena. They can't occur in linear systerns. Of course, a linear system, such as a linear differential equation, can have closed orbits – periodic solutions, but they won't be isolated. If $x(t)$ is a periodic solution, then so is $\alpha x(t)$ for any constant $\alpha \neq 0$. Hence $x(t)$ is surrounded by a 'family' of closed orbits. Consequently, the amplitude of a linear oscillation is set entirely by its initial

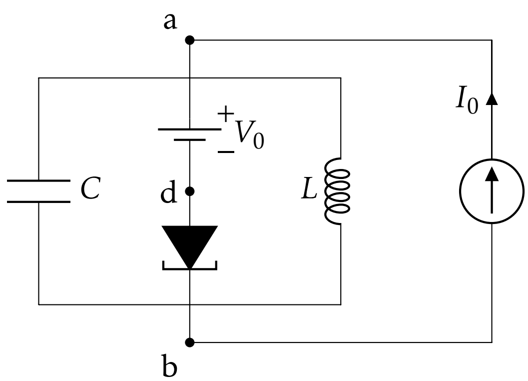

Figure 1: The Fitzhugh-Nagumo circuit used to model the nerve membrane. With a cubically nonlinear tunnel diode, $I = I_0 + F(V - V_0)$, $F(V) = \left(\frac{\alpha}{3}V^3 - \beta V\right)$, it is described by van der Pol equation.

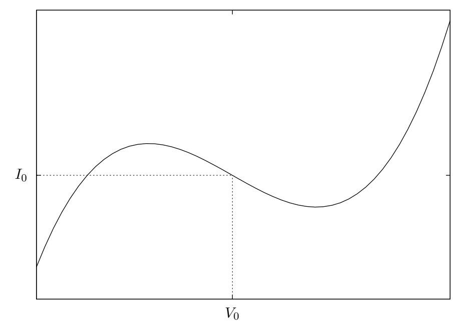

Figure 2: $I(V)$ for a tunnel diode. Note the "negative resistance", $\mathrm{d}I/\mathrm{d}V < 0$, for $V \sim V_0$.

conditions. Any slight disturbance to the amplitude will persist forever. In contrast, limit cycle oscillations are determined by the structure of the system itself.

Limit cycles are only possible in systems with dissipation. System that conserve energy do not have isolated closed trajectories.

(a) (40 points) Consider the IVP Eq. (1) with the initial conditions $x(0) = 1$, $\dot{x}(0) = 0$. Consider separately two cases: week nonlinearity, $\varepsilon = 0.1$, and strong nonlinearity, $\varepsilon = 10$. Solve Eq. (1) using Julia's `OrdinaryDiffEq` package. Consider the range of the independent variable t that covers about five periods of limit cycles. (The periods are different for week and strong nonlinearity cases.) On two different figures plot the phase trajectories $\dot{x}(x)$. Provide the legend, grid, title, axes labels for each of your graphs.

Are the limit cycles of the van der Pol equation stable or unstable? Describe (in the README.md file of your git project) your reasoning as well as the qualitative difference between limit cycles for week and strong nonlinearity.

Git

3. (15 points)

Clean the cells of your jupyter notebook and save it.

Create a git repository for hw05. Check your notebook, project files, .gitignore file into the repository. Provide meaningful commit messages.

Gitlab

4. (15 points)

   Create an empty Gitlab project called **hw05** (name it exactly as shown). At this step un-check the box “Create README.md file”.

   Push the content of your git repository to Gitlab’s **hw05** project

   On the Gitlab “side” create README.md file.

   Pull the README.md file to your local git repository to synchronize your local and remote repositories.

   I have synchronized the contents of my local and remote repositories that I created for hw05 assignment

   Sign and date here: ____________________

   Share the project with the instructor and grant him **Reporter** privileges.