Name: _____

Date: _____

Collaborators: _____

(If applicable, the collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	5	30	15	15	65
Score:					

Instructor/grader comments:

Use jupyter notebook interface to write the code for this homework assignment. Create a directory for the assignment (say **hw04**); change to that directory and work in there. When you start julia inside the folder (for the first time only), activate the project and add packages that you will use (IJulia, PyPlot, etc.). Place all code for the assignment in a single notebook file (call it e.g. **hw04.ipynb**).

1. (5 points) I read the article What is a plain English explanation of Big O notation? which was a part of Homework 3 assignment.

   Sign and date here: _____

2. Consider the IVP for a single first order differential equation:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t,y), \quad a \le t \le b, \quad y(a) = \alpha. \tag{1}$$

   A Runge-Kutta method for solving Eq. (1), known as midpoint method, is the following algorithm:

$$\begin{aligned} k_1 &= h f(t_n, y_n), \\ k_2 &= h f(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}), \\ y_{n+1} &= y_n + k_2, \end{aligned} \tag{2}$$

   where $n = 1, \ldots, k-1$, $t_n = a + (n-1)h$, h is the integration step, $h = (b-a)/(k-1)$.

   (a) (15 points) write a generic function `myrk2(fun, a, b, m, y0)` that accepts as the arguments the function of two variables, $fun(t,y)$ (the right-hand side of Eq. (1)), the integration limits a and b, the number of nodes m, and the initial value $y(a) = y0$, uses the Runge-Kutta midpoint method, and returns two vectors, t and y, where $y_n = y(t_n)$, $n = 1, \ldots, m$ is the solution of the IVP at t_i.

   (b) (15 points) Consider the IVP,

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y, \quad 0 \le t \le 5, \quad y(0) = 1, \tag{3}$$

   with the exact solution

$$y_{\mathrm{ex}}(t) = e^t. \tag{4}$$

   Solve IVP Eq. (3) using your function `myrk2` for $k = 8, 16, 32, \ldots, 2048, 4096$ (i.e. $k = 2^{i+3}$, $i = 1, \ldots, 9$. On the same figure plot your solutions for $k = 16$ and the exact solution. Provide the legend, grid, title, axes labels for your graph.

Find the global error of your solutions defined as

$$\Delta(h_k) = \left| y_k - y_{\text{ex}}(b) \right|. \tag{5}$$

In a different figure plot Δ vs. the integration step h. By visual inspection determine the order of accuracy of the midpoint Runge-Kutta method. ((Use the appropriate style of plot axes.) Provide the legend, grid, title, axes labels for your graph. Describe your reasoning and the result of your numerical experiment in the README.md file of your git project.

Git

3. (15 points)

   Clean the cells of your jupyter notebook and save it.

   Create a git repository for hw04. Check your notebook, project files, .gitignore file into the repository. Provide meaningful commit messages.

Gitlab

4. (15 points)

   Create a Gitlab project called **hw04** (name it exactly as shown).

   Push the content of your git repository to Gitlab's **hw04** project

   On the Gitlab "side" create README.md file.

   Pull the README.md file to your local git repository to synchronize your local and remote repositories.

   I have synchronized the contents of my local and remote repositories that I created for hw03 assignment

   Sign and date here: _____

   Share the project with the instructor and grant him **Reporter** privileges.