Name: ________________________________

Date: ________________________________

Collaborators: ____

(If applicable, the collaborators submit their individually written assignments together)

Question:	1	2	3	4	Total
Points:	30	30	15	20	95
Score:					

Instructor/grader comments:

Use jupyter notebook interface to write the code for this homework assignment. Create a directory for the assignment (say **hw03**); change to that directory and work in there. When you start julia inside the folder (for the first time only), activate the project and add packages you will use (IJulia, PyPlot, etc.). Place all code for the assignment in a single notebook file (call it e.g. **hw03.ipynb**).

1. Simpson's quadrature used for numerical evaluation of integrals, is as following:

$$\int\limits_a^b f(x)\,\mathrm{d}x \approx \frac{h}{3}\left(f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + \ldots + 4f(x_{n-1}) + f(x_n)\right) + O(h^p), \quad (1)$$

   where

$$h = \frac{b-a}{n-1}, \quad (2)$$

   and p is the order of accuracy of the quadrature. The number of nodes, n, in Eq. (1) must be an odd number (larger than 1).

   (a) (15 points) write a function `mysimpsons(fun, a, b, n)` that accepts the integrand, the integration limits, and the number of nodes, and returns the approximate numerical value of the integral. Test your function for 2-3 elementary integrals.

   (b) (15 points) Consider the integral,

$$\int_0^{\pi} \sin(x)\,\mathrm{d}x = 2. \quad (3)$$

   Evaluate this integral numerically for multiple values of h using your function `mysimpsons(fun, a, b, n)`, plot the errors of the result, $\Delta(h)$, in appropriate axis (linear, semilog, loglog, etc.), and determine the order of accuracy, p, of the Simpson's quadrature. Describe your reasoning and the result of your numerical experiment in the README.md file of your git project.

2. Legendre polynomials is a particular form of *special functions* that have important applications in numerical analysis and in physics. They are named after a French mathematician Adrien-Marie Legendre who discovered them in 1782.

   Legendre polynomial $P_n(x)$ satisfy the following recurrence relations:

$$(n+1)\,P_{n+1}(x) = (2n+1)\,x\,P_n(x) - n\,P_{n-1}(x), \quad n = 1, 2, \ldots; \quad P_0(x) = 1, P_1(x) = x. \quad (4)$$

The Legendre polynomials form *an orthogonal system* over the interval $[-1,1]$, i.e.

$$\int_{-1}^{1} P_m(x)\,P_n(x)\,\mathrm{d}x = 0, \quad \text{if } n \neq m. \tag{5}$$

(a) (10 points) Write a function `mylegendre(n, x)` that uses the recurrence relations (4) to calculate the value of Legendre polynomial $P_n(x)$

(b) (5 points) Use your function, `mylegendre(n, x)`, to plot, in one figure, the graphs of $P_1(x)$, $P_3(x)$, and $P_5(x)$. Provide the legend, grid, title, axes labels.

(c) (15 points) Use your implementation of Simpson's quadrature to verify numerically the validity of Eq. (5) for $n = 3$ and $m = 4, 5, 6$. Evaluate each integral in its own notebook cell. Chose the number of nodes such that the numerical error of calculating of the integral Eq. (5) is less than 10^{-6}. Describe the result of your numerical experiment in the README.md file of your git project.

Git

3. (15 points)

   Clean the cells of your jupyter notebook and save your jupyter notebook.

   Create a git repository for hw03. Check your notebook, project files, .gitignore file into the repository. Provide meaningful commit messages.

Gitlab

4. (20 points)

   Create a Gitlab project called **hw03** (name it exactly as shown).

   Push the content of your git repository to Gitlab's **hw03** project

   On the Gitlab "side" create README.md file.

   Pull the README.md file to your local git repository to synchronize your local and remote repositories.

   I have synchronized the contents of my local and remote repositories that I created for hw03 assignment

   Sign and date here: ________________________________

   Share the project with the instructor and grant him **Reporter** privileges.