1 (*) Uniform solid disk $\Rightarrow I = \frac{1}{2} M r^2$

(**) Constant force $\Rightarrow$ constant torque

$$\tau = F \cdot r$$

(***) Constant torque $\Rightarrow$ constant angular acceleration

$$\tau = I \cdot \varepsilon \implies \varepsilon = \frac{\tau}{I}$$

$$I = \frac{1}{2} \cdot 100\,kg \cdot 4\,m^2 = 200\ kg \cdot m^2$$

$$\tau = 50\,N \cdot 2\,m = 100\ N \cdot m = 100\ \frac{kg \cdot m^2}{s^2}$$

$$\varepsilon = \frac{100\ \frac{kg \cdot m^2}{s^2}}{200\ kg \cdot m^2} = 0.5\ s^{-2}$$

(****) Constant angular acceleration $\Rightarrow$ linear in time angular velocity

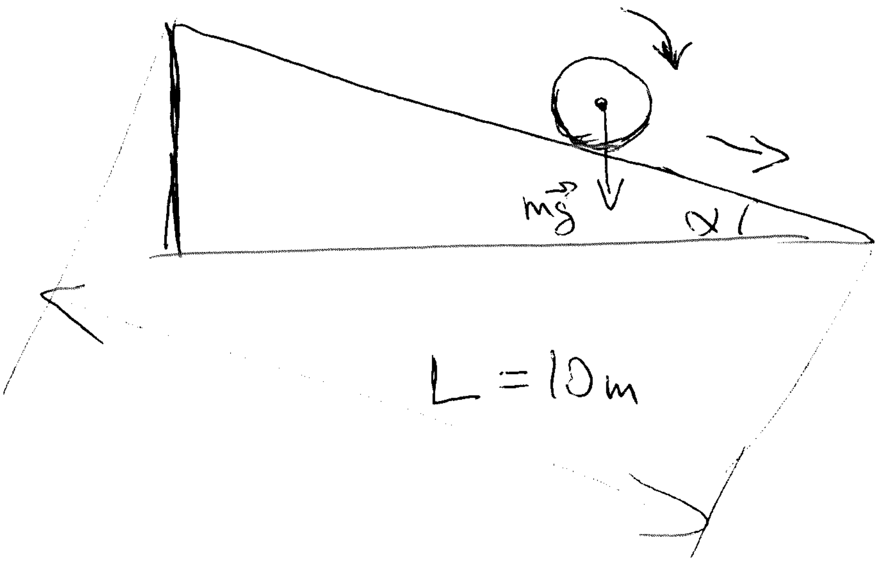
$$\omega = \varepsilon t = 0.5\,s^{-2} \cdot 2\,s = \underline{1\ s^{-1}}$$

Kinetic energy of rotation

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot 200\,kg \cdot m^2 \cdot \underline{1}\,s^{-2}$$

$$= \underline{\underline{100\ J}}$$

#2



Rolling with constant acceleration (both angular and linear)

Linear acceleration:

$$L = \frac{1}{2} a t^2$$

$$a = \frac{2L}{t^2} = \frac{2 \cdot 10\,m}{4\,s^2} = 5\,m/s^2$$

On the other hand, angular acceleration of the rolling cylinder:

$\varepsilon = \frac{\tau}{I}$, where $\tau = mgr\sin\alpha$ – torque created by gravity, I – unknown moment of inertia

$$\varepsilon = \frac{a}{r} \longrightarrow I = \frac{\tau}{\varepsilon} = \frac{\tau \cdot r}{a} = \frac{g}{a} \cdot mr^2 \sin\alpha$$

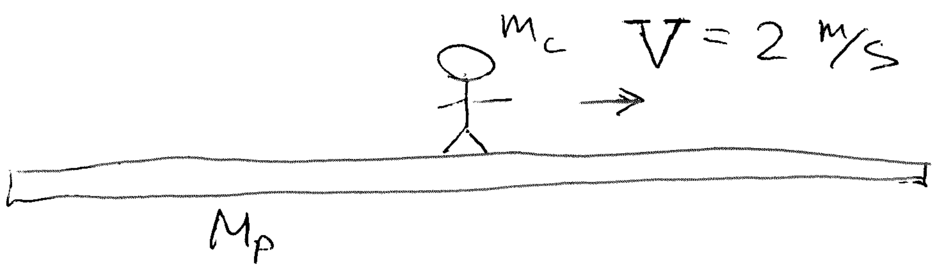
$$= 2 \cdot mr^2 \cdot \frac{1}{2} = mr^2$$

cylinder is hollow!

$$I = mr^2 = 1\,kg \cdot 36 \times 10^{-4}\,m^2 =$$

$$= 3.6 \times 10^{-3}\,kg \cdot m^2$$

3



No friction → conservation of linear momentum

V_c – child's velocity w/r to lake

V_p – plank velocity w/r to lake

$$m_c V_c - M_p V_p = 0 \Rightarrow V_c = \frac{M_p}{m_c} V_p$$

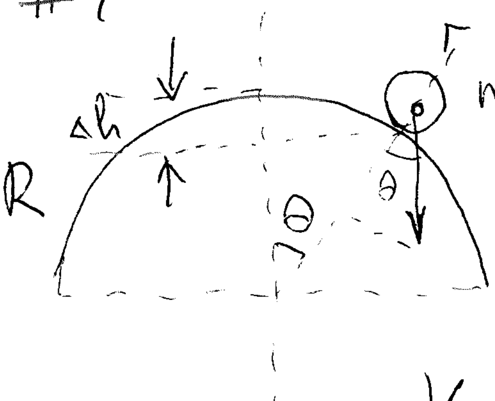
The velocity of the child w/r to the plank:

$$V \equiv V_c + V_p = V_p + \frac{M_p}{m_c} V_p$$

$$V_p = \frac{V}{1 + \frac{M_p}{m_c}} = \frac{2 \text{ m/s}}{4} = 0.5 \text{ m/s}$$

$$V_c = \frac{M_p}{m_c} \frac{V}{1 + \frac{M_p}{m_c}} = \frac{V}{1 + \frac{m_c}{M_p}} = \frac{3}{4} \cdot 2 \text{ m/s} = 1.5 \text{ m/s}$$

#4



(*) Kinetic energy of the rolling uniform sphere (pumkin)

$$K = \frac{1}{2} m v^2 + \frac{1}{2} I_{sp} \omega^2$$

$$I_{sp} = \frac{2}{5} m r^2 \qquad \omega = \frac{v}{r}$$

$$K = \frac{1}{2} m v^2 + \frac{\cancel{1}}{\cancel{2}} \cdot \frac{\cancel{2}}{5} \cdot m \cancel{r^2} \frac{v^2}{\cancel{r^2}} = \frac{7}{10} m v^2$$

(**) Conservation of energy:

$$\left.\begin{array}{l} \cancel{m} g \Delta h = \frac{7}{10} \cancel{m} v^2 \\ \Delta h = R - R\cos\theta \end{array}\right\} \Rightarrow v^2 = \frac{10}{7} g R (1-\cos\theta)$$

(***) Normal reaction of the cup:

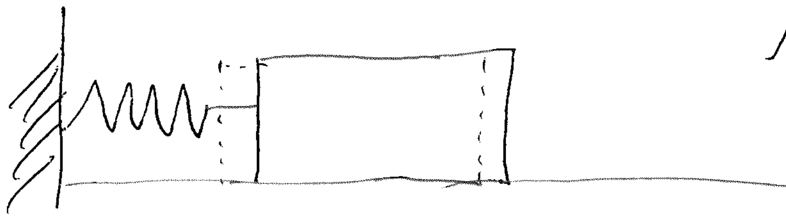
$$N = \frac{m v^2}{R} - m g \cos\theta$$

The pumkin leaves the cap when $N = 0$

$$\frac{\cancel{m} v^2}{R} = \cancel{m} g \cos\theta \Rightarrow \frac{10}{7} \cancel{g} (1-\cos\theta) = \cancel{g} \cos\theta \Rightarrow$$

$$\frac{10}{7} = \frac{17}{7} \cos\theta \Rightarrow \cos\theta = \frac{10}{17} \Rightarrow \theta = \dots$$

#5



$\mu_k = \cancel{0.36}\ 0.18$

(*) No friction → conservation of energy

$\frac{1}{2} k \Delta x^2 = \frac{1}{2} m v_0^2$

$\Delta x = 0.1\,m$
$v_0 = 1\,m/s$

(**) With friction

$\frac{1}{2} k \Delta x^2 = \frac{1}{2} m v_1^2 + F_{fr} \cdot \Delta x$

$F_{fr} = m g \mu_k$

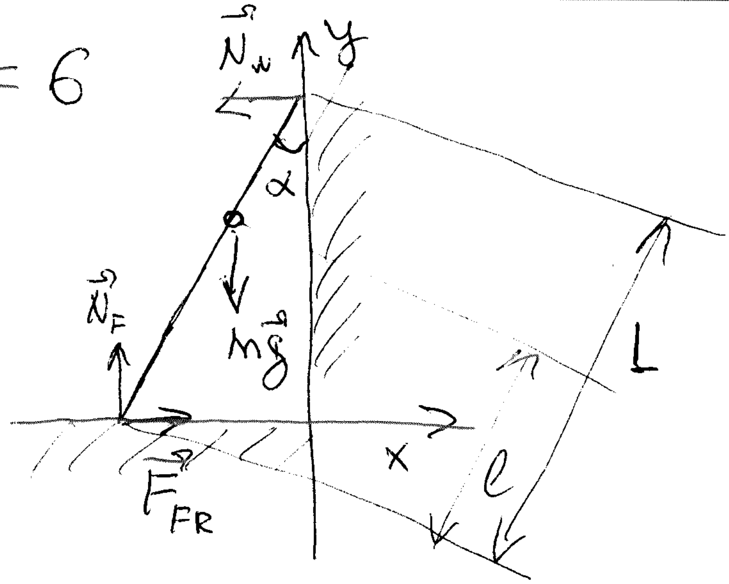
$\frac{1}{2} m v_1^2 = \underbrace{\frac{1}{2} k \Delta x^2}_{= \frac{1}{2} m v_0^2} - m g \Delta x \mu_k$

$\frac{1}{2} \cancel{m} v_1^2 = \frac{1}{2} \cancel{m} v_0^2 - \cancel{m} g \Delta x \mu_k$

$v_1 = \sqrt{v_0^2 - 2 g \Delta x \mu_k} = \sqrt{1\,m^2/s^2 - 2 \times 10 \times 0.1 \times 0.18}$

$= 0.8\,m/s$

6



$$\vec{N}_F + \vec{N}_w + \vec{F}_{FR} + m\vec{g} = 0$$

$$x:\ -F_{FR} + N_w = 0$$

$$N_w = F_{FR} = N_F \cdot \mu_s$$

$$y:\quad N_F - mg = 0 \Rightarrow N_F = mg \Rightarrow N_w = \mu_s mg$$

$$\tau:\quad mg\ell\sin\alpha - N_w L\cos\alpha = 0$$

$$\cancel{mg}\,\ell\sin\alpha = \mu_s \cancel{mg} \cdot L\cos\alpha$$

$$\ell = L \cdot \mu_s \cdot \frac{\cos\alpha}{\sin\alpha} = L/4$$