#1

$$\vec{a} = (1, 1)$$

$$\vec{b} = (3, 0)$$

$$\vec{c} = (0, 1)$$

(a) $\vec{a} + 2\vec{b} + 3\vec{c} =$

$= (1,1) + 2(3,0) + 3(0,1) =$

$= (1,1) + (6,0) + (0,3)$

$= (7,4)$

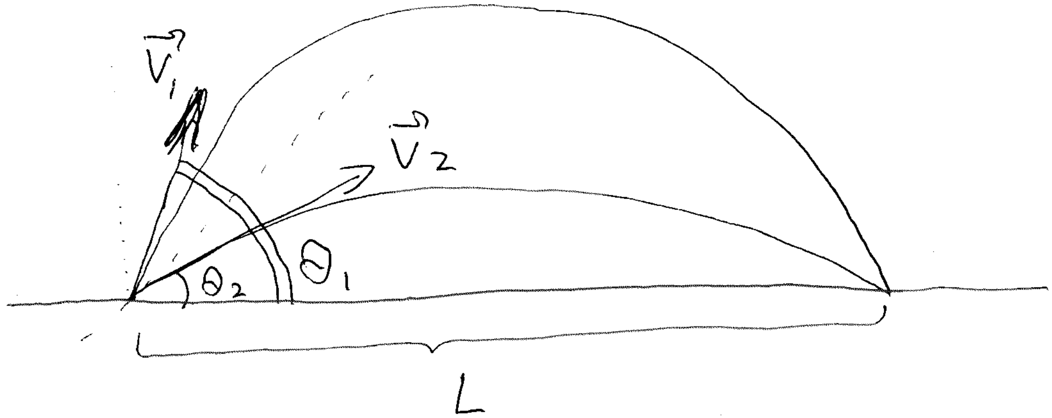
(b) $2\vec{a} - \vec{b} - 2\vec{c} =$

$= 2(1,1) - (3,0) - 2(0,1)$

$= (2,2) - (3,0) - (0,2) =$

$= (-1, 0) \equiv -(1,0)$

#2



$|\vec{V}_1| = |\vec{V}_2| = V$; $L_1 = L_2 = L$

$$\left.\begin{aligned} L_1 &= \frac{V^2}{g}\sin(2\theta_1) \\ L_2 &= \frac{V^2}{g}\sin(2\theta_2) \end{aligned}\right\} \Rightarrow \sin 2\theta_1 = \sin 2\theta_2 \quad \theta_1 \neq \theta_2 \;!$$

Therefore $2\theta_2 = \pi - 2\theta_1$

$$\boxed{\theta_2 = \frac{\pi}{2} - \theta_1}$$

(a) $\theta_1 = 65°$
$\theta_2 = 25°$

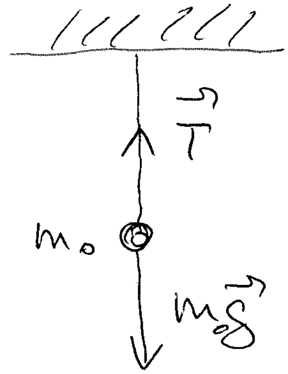
(b) Time to fly: $V_{1x} = V\cos\theta_1$; $V_{2x} = V\cos\theta_2$

$$t_1 = \frac{L_1}{V_{1x}} = \frac{V^{\cancel{2}}}{g}\frac{2\sin\theta_1\cos\theta_1}{\cancel{V}\cos\theta_1} = \frac{2V}{g}\sin\theta_1$$

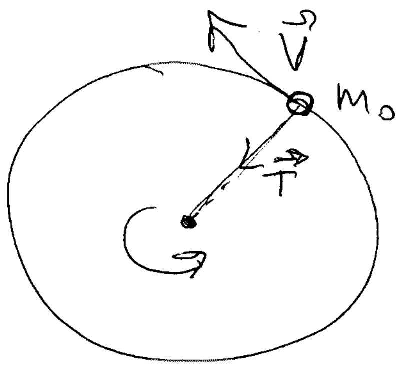
$$t_2 = \frac{L_2}{V_{2x}} = \dots = \frac{2V}{g}\sin\theta_2 \quad \left(= \frac{2V}{g}\cos\theta_1\right)$$

$$\Delta t = t_1 - t_2 = \frac{2V}{g}\left(\sin\theta_1 - \sin\theta_2\right) = \dots\dots$$

3



$$T_{max} = m_0 g$$



$$T = \frac{m_0 v^2}{r}$$

$$\frac{m_0 v_{max}^2}{r} = T_{max} = m_0 g$$

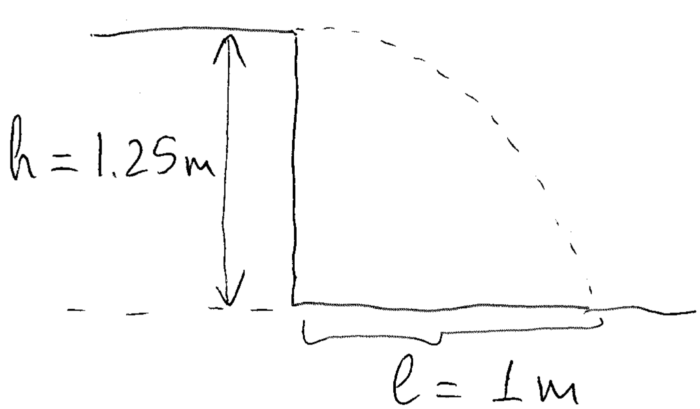
$$\frac{m_0 v_{max}^2}{r} = m_0 g$$

$$v_{max}^2 = \sqrt{gr} = \sqrt{10\,m/s^2 \cdot 0.1\,m} = 1\,m/s$$

Range:

$$0 < v \leq v_{max}$$

4



Time to fall:

$$h = \frac{1}{2} g t^2$$

$$t^2 = \frac{2h}{g}$$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2.5\,m}{10\,m/s^2}} = \frac{1}{2}\,s$$

Horizontal component of mug's velocity (the same at all times)

$$V_x = \frac{l}{t} = \frac{1\,m}{1/2\,s} = \boxed{2\,m/s}$$

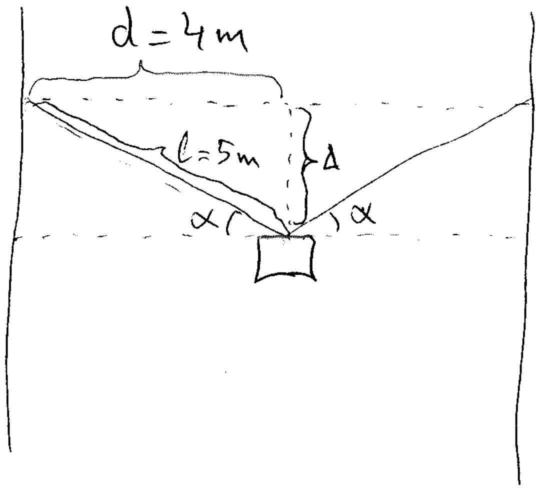
Vertical component of mug's velocity just before the end of the flight

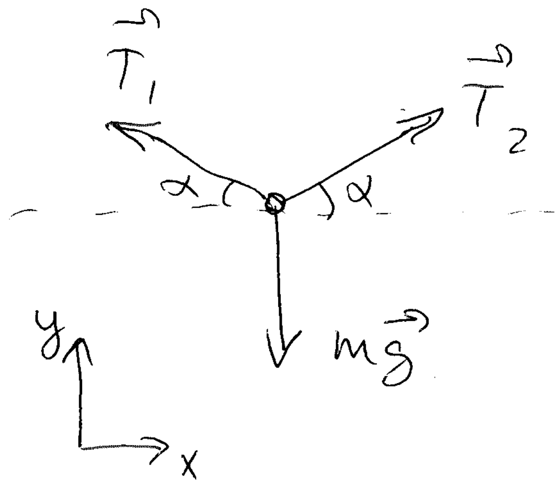
$$V_y = gt = 10\,m/s^2 \cdot \frac{1}{2}\,s = 5\,m/s$$

Magnitude of mug's velocity

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{29}\,m/s = \ldots$$

#5





$$\Delta = \sqrt{l^2 - d^2} = 3\,m$$

$$\sin\alpha = \frac{\Delta}{l} = \frac{3}{5}$$

$$\vec{T}_1 + \vec{T}_2 + m\vec{g} = 0$$

$$|\vec{T}_1| = |\vec{T}_2| = T$$

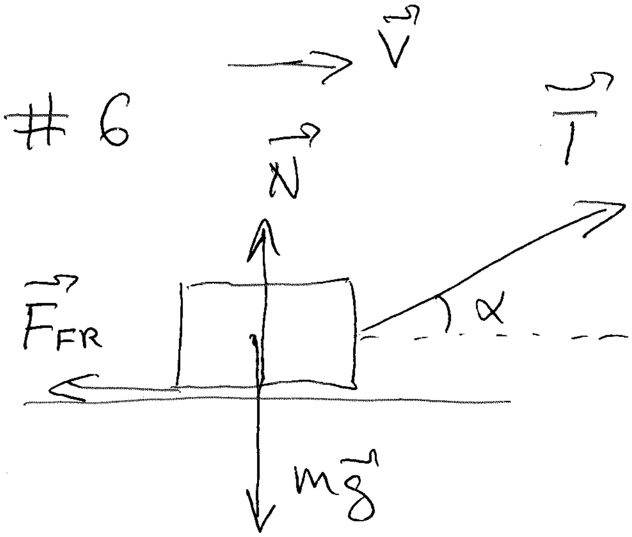
(Symmetry)

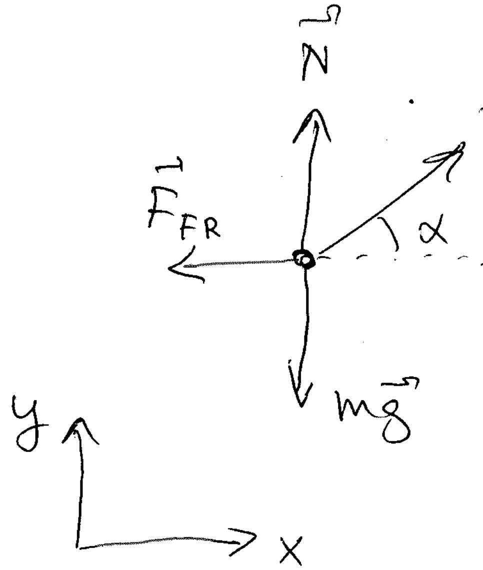
$$y: \quad -mg + 2T\sin\alpha = 0$$

$$T = \frac{mg}{2\sin\alpha} = \frac{20\,kg \cdot 10\,m/s^2}{2 \cdot 3/5}$$

$$= \frac{500}{3}\,N = \ldots$$

6





$$\vec{V} = \text{const} \implies \vec{a} = 0$$

$$\vec{F}_{FR} + m\vec{g} + \vec{N} + \vec{T} = 0$$

X: $$-F_{FR} + T\cos\alpha = 0$$

$$\cos\alpha = \frac{F_{FR}}{T} = \frac{20N}{25N} = \frac{4}{5}$$

(a) $$\alpha = \arccos\left(\frac{4}{5}\right) = \ldots\ldots$$

$$\sin\alpha = \frac{3}{5}$$

y: $$-mg + N + T\sin\alpha = 0$$

$$N = mg - T\sin\alpha = 10\,\text{kg}\cdot 10\,\text{m/s}^2 - 25N\cdot\frac{3}{5}$$

(b) $$= \underline{\underline{85N}}$$