

312. Homework 4

R. Côté

Due: Thursday, February 26, 2004

PROBLEM 1: Bessel functions. In Arfken and Weber, do problems 11.1.2 and 11.1.6.

PROBLEM 2: More Bessel functions. In Arfken and Weber, do problems 11.4.6 and 11.4.7.

PROBLEM 3: From the definition of Neumann functions,

$$N_\nu(x) = \frac{\cos \pi\nu J_\nu(x) - J_{-\nu}(x)}{\sin \pi\nu},$$

show that

- (i) $N_\nu(x)$ is indeterminate for $\nu = n$ (an integer).
- (ii) Use L'Hospital rule to show that the leading term (for small x) of $N_0(x)$ is given by

$$N_0(x) = \frac{2}{\pi}(\ln x + \gamma - \ln 2) + \mathcal{O}(x^2).$$

- (iii) For an arbitrary n (integer), show that

$$N_n(x) = -\frac{(n-1)!}{\pi} \left(\frac{2}{x}\right)^n + \cdots + \frac{2}{\pi} \left(\frac{x}{2}\right)^n \frac{1}{n!} \ln\left(\frac{x}{2}\right) + \cdots$$