

312. Homework 3

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Due: Thursday, February 12, 2004

PROBLEM 1: Find the kind of boundary conditions that will result in orthogonal eigenfunctions for the Sturm-Liouville differential equation

$$\mathcal{L}y(x) + \lambda y(x) = 0 ,$$

where

$$\mathcal{L} \equiv \frac{d}{dx} \left[p(x) \frac{d}{dx} \right] + q(x) .$$

All quantities are real. (Hint: use two different sets of eigenvalues and eigenfunctions, and get the orthogonality condition over the appropriate range $a \leq x \leq b$.)

PROBLEM 2: Orthogonal functions and Green's functions. In Arfken and Weber, do problems 9.3.3, 9.5.3, and 9.5.13.

PROBLEM 3: Gamma functions. In Arfken and Weber, do problems 10.1.6 and 10.1.11.

PROBLEM 4: More gamma functions.

(i) Show that

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin z\pi} .$$

(Hint: use Weierstrass infinite-product definition for $\Gamma(z)$.)

(ii) Derive Stirling series for $\ln(z!)$ (for large z) using the Euler-Maclaurin formula for evaluating a definite integral, and the appropriate Bernoulli numbers.