Physics 1202: Lecture 17
Today's Agenda

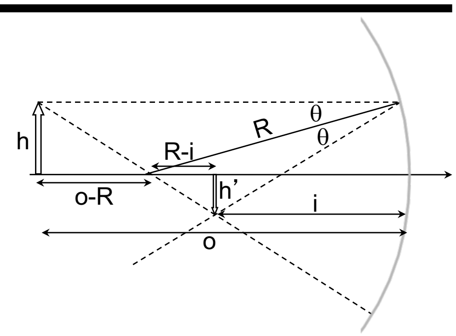
- Announcements: Team problems today
  - Team 10, 11 & 12: this Thursday
- **Homework #8: due Friday**
- **Midterm 2:**
  - **Tuesday April 10**
  - **Office hours if needed (M-2:30-3:30 or TH 3:00-4:00)**
- Chapter 26:
  - Geometrical Optics
  - Mirrors equation
  - Refraction

26

Reflection

$$\theta_i = \theta_r$$

Mirrors



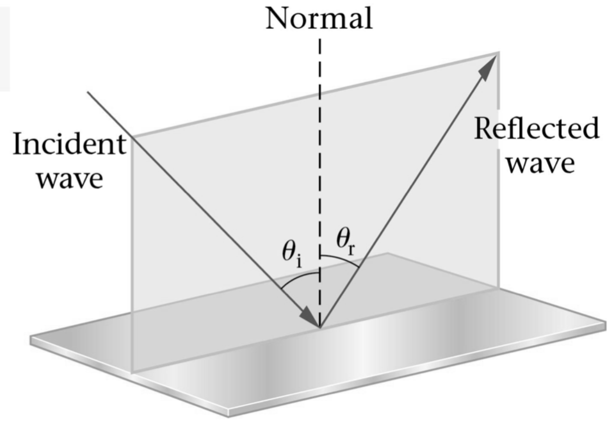
Lecture 17

26-1 The Reflection of Light

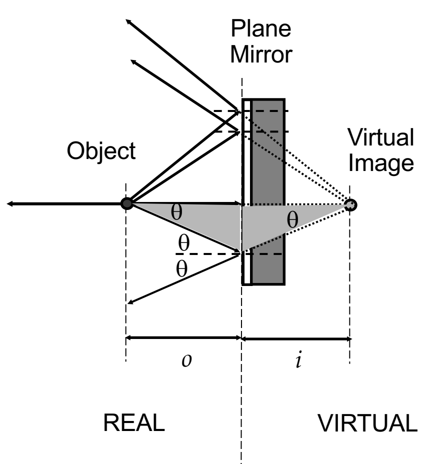
- The law of reflection states that the angle of incidence equals the angle of reflection:

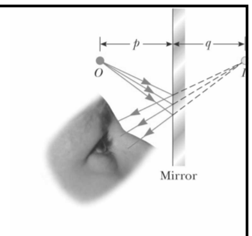
Law of Reflection

$\theta_r = \theta_i$



Flat Mirror

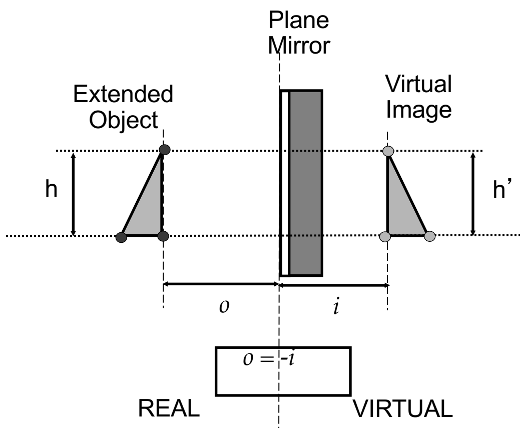




$o = -i$

Flat Mirror

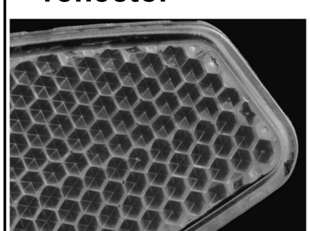
Images of Extended Objects

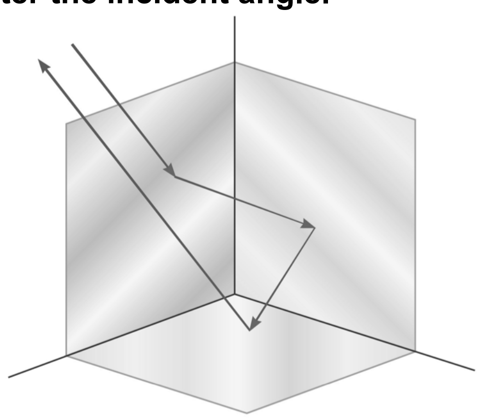


Magnification:
M = h'/ h = 1

26-2 Applications of Plane Mirror

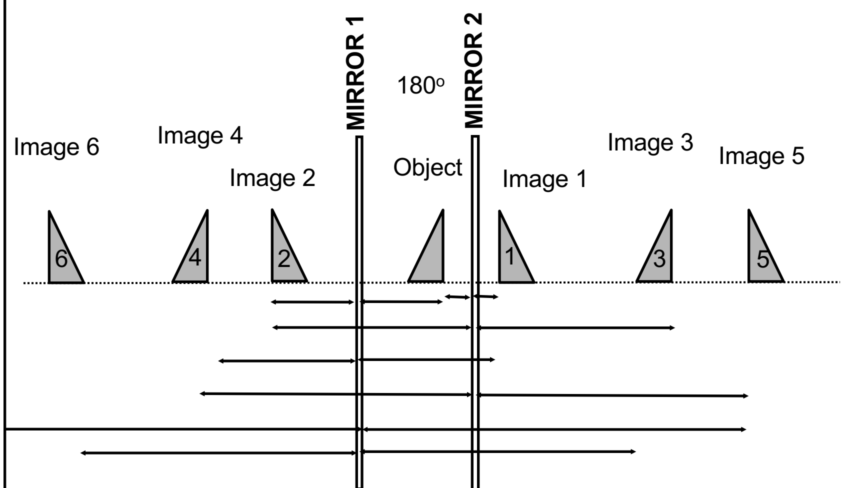
- **A corner reflector reflects light parallel to the incident ray, no matter the incident angle.**
- **E.g. bicycle reflector**





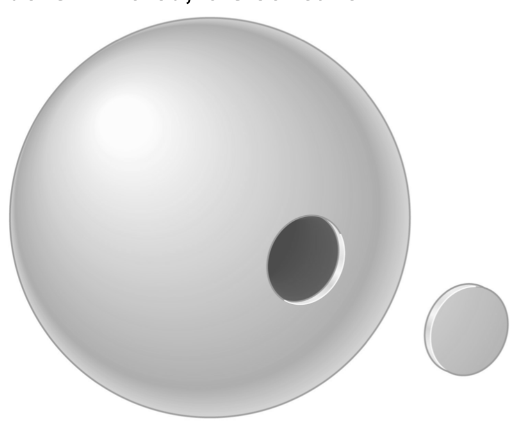
(a)

Multiple Reflection



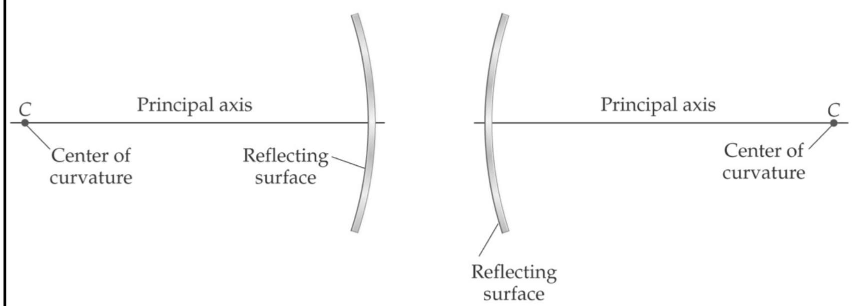
26-3 Spherical Mirrors

- A spherical mirror has the shape of a section of a sphere.
  - If the outside is mirrored, it is convex
  - if the inside is mirrored, it is concave.



26-3 Spherical Mirrors: definitions

- Spherical mirrors have
  - a central axis (a radius of the sphere) and a
  - center of curvature (the center of the sphere)

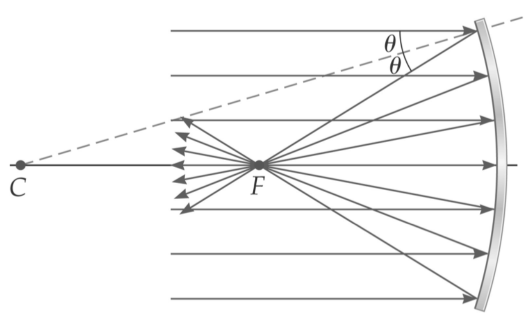


Concave mirror

Convex mirror

26-3 Concave Spherical Mirrors

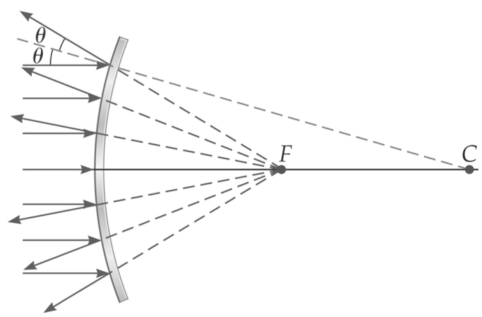
- Consider parallel rays
- They hit a spherical mirror
- They come together at the focal point



- This is a ray diagram for finding the focal point of a concave mirror.

26-3 Convex Spherical Mirrors

- Consider parallel rays
- They hit a spherical mirror
- They appear to have come from the focal point, if the mirror is convex



26-3 Spherical Mirrors

- For a concave mirror, the focal length is postive, as the rays go through the focal point.

Focal Length for a Concave Mirror of Radius *R*

$$f = \frac{1}{2}R \qquad \text{26-3}$$

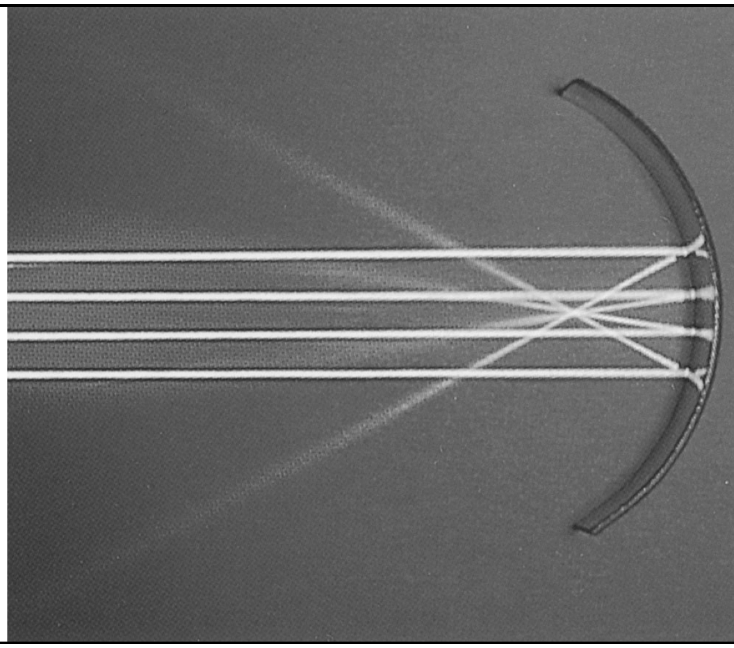
SI unit: m

- For a convex mirror, the focal length is negative, as the rays do not go through the focal point.

Focal Length for a Convex Mirror of Radius *R*

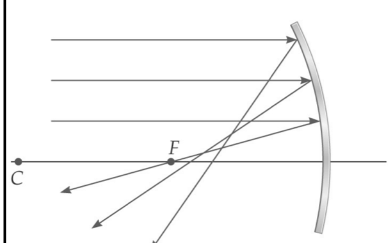
$$f = -\frac{1}{2}R \qquad \text{26-2}$$

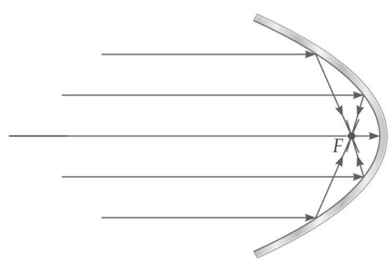
SI unit: m



26-3 Hyperbolic Mirrors

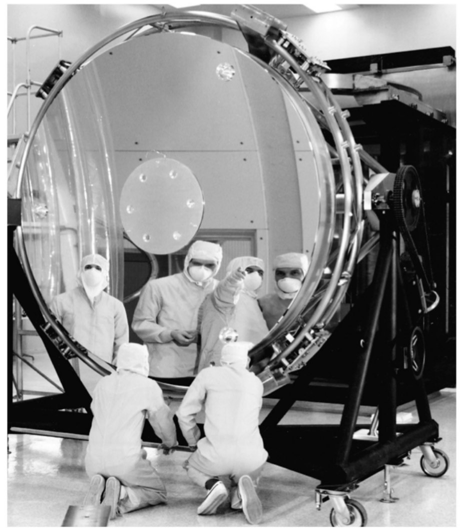
- For spherical mirrors
- Assumption that the rays do not hit the mirror very far from the principal axis
- Otherwise, the image is blurred;
  - this is called spherical aberration,
- Remedied by using a parabolic mirror instead.





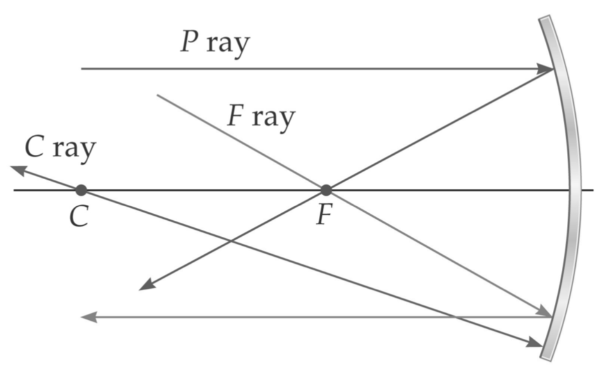
26-3 Spherical Mirrors

- When the Hubble Space Telescope was first launched, its optics were marred by spherical aberration. This was fixed with corrective optics.



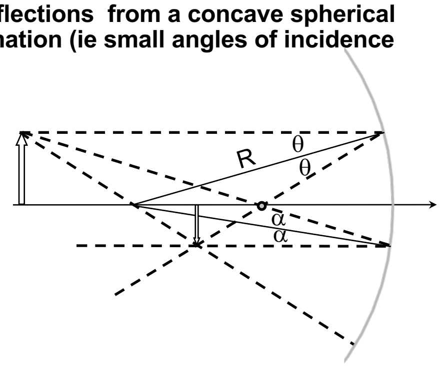
26-4 Ray Tracing & Mirror Equation

- We use three principal rays in finding the image produced by a concave mirror.
  - The parallel ray (*P* ray) reflects through the focal point.
  - The focal ray (*F* ray) reflects parallel to the axis.
  - The center-of-curvature ray (*C* ray) reflects back along its incoming path.



Concave Spherical Mirrors

- **We start by considering the reflections from a concave spherical mirror in the paraxial approximation (ie small angles of incidence close to a single axis):**



• First draw a ray (light blue) from the tip of the arrow through the center of the sphere. This ray is reflected straight back since the angle of incidence = 0.

• Now draw a ray (white) from the tip of the arrow parallel to the axis. This ray is reflected with angle θ as shown.

• Note that the two rays intersect in a point, suggesting an inverted image.
• To check this, draw another ray (green) which comes in at some angle α that is just right for the reflected ray to be parallel to the optical axis.
• Note that this ray intersects the other two at the same point, as it must if an image of the arrow is to be formed there.
• Note also that the green ray intersects the white ray at another point along the axis. We will call this point the focal point (∘).

The Mirror Equation

- **We will now transform the geometric drawings into algebraic equations:**

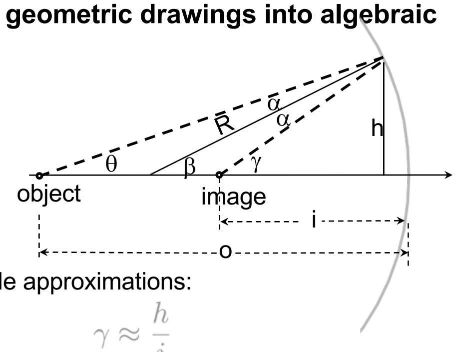
from triangles,

$$\beta = \alpha + \theta$$

$$\gamma = 2\alpha + \theta$$

eliminating α,

$$\gamma = 2\beta - \theta$$



Now we employ the small angle approximations:

$$\theta \approx \frac{h}{o} \qquad \beta \approx \frac{h}{R} \qquad \gamma \approx \frac{h}{i}$$

Plugging these back into the above equation relating the angles, we get:

$$\frac{1}{i} = \frac{2}{R} - \frac{1}{o}$$

Defining the focal length f = R/2,

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

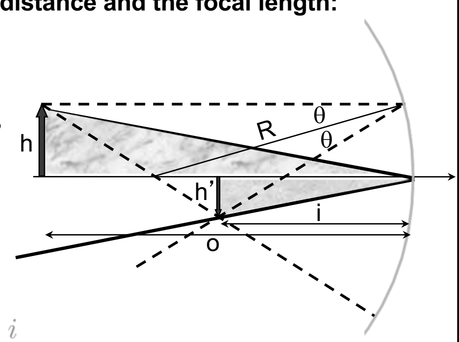
This eqn is known as the mirror eqn. Note that there is no mention of θ in this equation. Therefore, this eqn works for all θ, ie we have an image!

Magnification

- **We have derived the mirror eqn which determines the image distance in terms of the object distance and the focal length:**

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

- What about the size of the image?
  - How is h' related to h??
  - From similar triangles:



$$\frac{h}{o} = \frac{h'}{i}$$

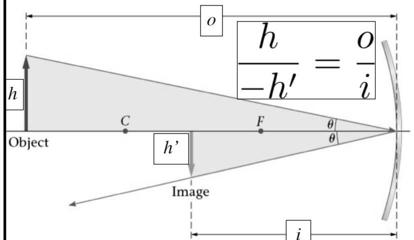
$$\frac{h'}{h} = \frac{i}{o}$$

Now, we can introduce a sign convention. We can indicate that this image is inverted if we define its magnification M as the negative number given by:

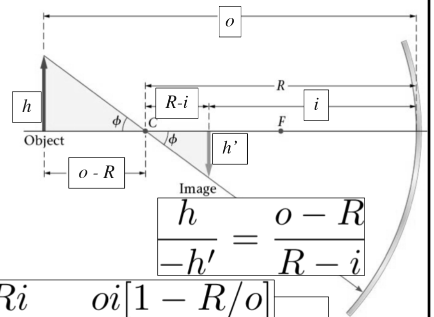
$$M = -\frac{i}{o}$$

26-4 Mirror Equation: from the book

- **Derivation of mirror equation using the ray diagrams**



$$\frac{h}{-h'} = \frac{o}{i}$$



$$\frac{h}{-h'} = \frac{o-R}{R-i}$$

- **equating**

$$\frac{o}{i} = \frac{o-R}{R-i} \Rightarrow 1 = \frac{oi - Ri}{Ro - oi} = \frac{oi[1-R/o]}{oi[R/i-1]}$$

$$\Rightarrow \frac{R}{i} - 1 = 1 - \frac{R}{o}$$

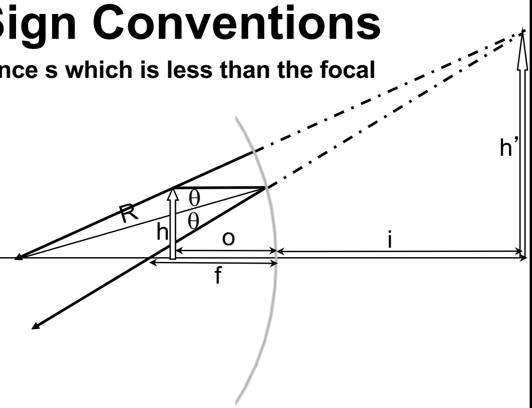
$$\Rightarrow \frac{R}{i} + \frac{R}{o} = 2$$

$$\Rightarrow \frac{1}{i} + \frac{1}{o} = \frac{2}{R} = \frac{1}{f}$$

More Sign Conventions

- **Consider an object distance s which is less than the focal length:**

Ray Trace:
- Ray through the center of the sphere (light blue) is reflected straight back.
- Ray parallel to axis (red) passes through focal point f.



- These rays diverge! ie these rays look they are coming from a point behind the mirror.
- We call this a virtual image, meaning that no light from the object passes through the image point.
- Proof left to student: This situation is described by the same mirror equations as long as we take the convention that images behind the mirror have negative image distances s'. ie:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

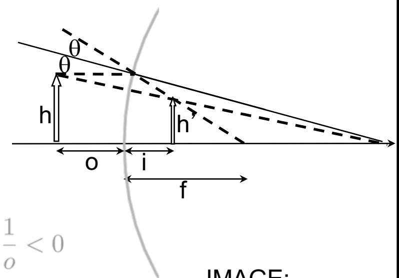
$$M = -\frac{i}{o}$$

In this case, $i < 0$, which leads to $M > 0$, indicating that the image is virtual ($i<0$) and not inverted ($M>0$).

Concave-Planar-Convex

- **What happens as we change the curvature of the mirror?**
  - **Plane mirror:**
    - **R = ∞** $\frac{1}{o} + \frac{1}{i} = 0$ ⇨ $i = -o$, $M = +1$

IMAGE:
virtual
upright (non-inverted)



  - **Convex mirror:**
    - R < 0

$$f = \frac{R}{2} < 0$$

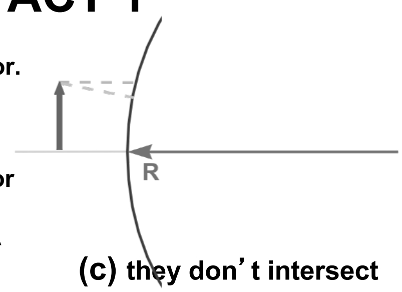
$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{o} < 0$$

⇨ $i < 0$, $M > 0$

IMAGE:
virtual
upright (non-inverted)

Lecture 17, ACT 1

- **Let's now consider a curved mirror. We start with CONVEX mirror.**
  - **Where do the rays which are reflected from the convex mirror shown intersect?**



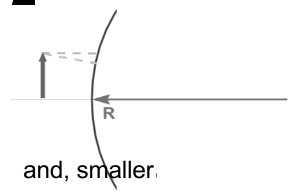
(a) to left of **(b) to right of** **(c) they don't intersect**

Lecture 17

Lecture 17, ACT 2

- What is the nature of the image of the arrow?

(a) Inverted and in front of the mirror
(b) Inverted and in back of the mirror
(c) Upright and in back of the mirror

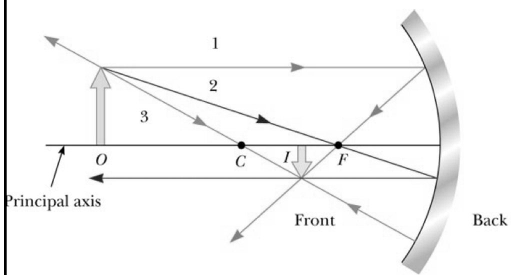


Lecture 17, ACT 3

- In order for a real object to create a real, inverted enlarged image,

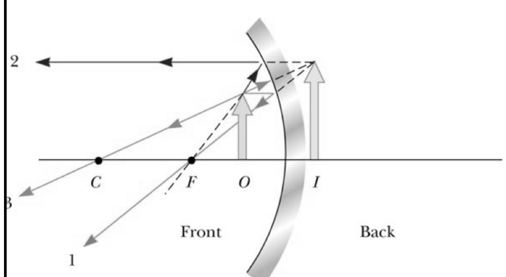
a) we must use a concave mirror.
b) we must use a convex mirror.
c) neither a concave nor a convex mirror can produce this image.

Diagram: Concave Mirror, o > R



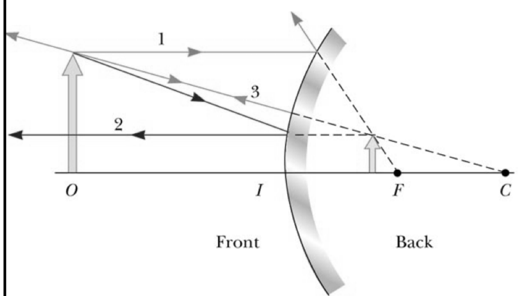
- The object is outside the center of curvature of the mirror
- The image is real
- The image is inverted
- The image is smaller than the object

Diagram: Concave Mirror, o < f



- The object is between the mirror and the focal point
- The image is virtual
- The image is upright
- The image is larger than the object

Diagram: Convex Mirror



- The object is in front of a convex mirror
- The image is virtual
- The image is upright
- The image is smaller than the object

Mirrors: image

TABLE 26-1 Imaging Characteristics of Convex and Concave Spherical Mirrors

Object location	Image orientation	Image size	Image type
CONVEX MIRROR			
Arbitrary	Upright	Reduced	Virtual
CONCAVE MIRROR			
Object location	**Image orientation**	**Image size**	**Image type**
Beyond *C*	Inverted	Reduced	Real
C	Inverted	Same as object	Real
Between *F* and *C*	Inverted	Enlarged	Real
Just beyond *F*	Inverted	Approaching infinity	Real
Just inside *F*	Upright	Approaching infinity	Virtual
Between mirror and *F*	Upright	Enlarged	Virtual

Mirrors: conventions

- **Here are the sign conventions for concave and convex mirrors:**

Focal Length

f is positive for concave mirrors.

f is negative for convex mirrors.

Magnification

m is positive for upright images.

m is negative for inverted images.

Image Distance

d_i is positive for images in front of a mirror (real images).

d_i is negative for images behind a mirror (virtual images).

Object Distance

d_o is positive for objects in front of a mirror (real objects).

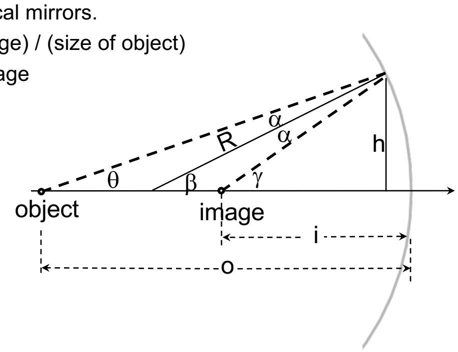
d_o is negative for objects behind a mirror (virtual objects).

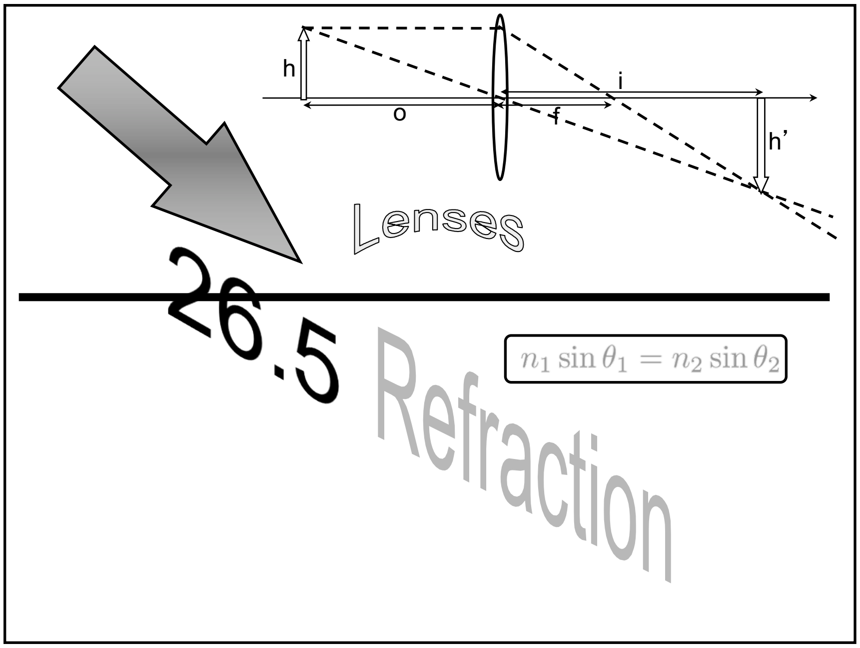
Mirror – Lens Definitions

- **Some important terminology we introduced last class,**
  - o = distance from object to mirror (or lens)
  - i = distance from mirror to image
    o positive, i positive if on same side of mirror as o.
  - R = radius of curvature of spherical mirror
  - f = focal length, = R/2 for spherical mirrors.
  - Concave, Convex, and Spherical mirrors.
  - M = magnification, (size of image) / (size of object)
    negative means inverted image

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

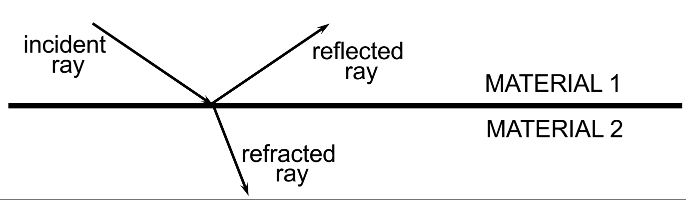
$$M = -\frac{i}{o}$$





EM wave at an interface

- What happens when light hits a surface of a material?
- Three Possibilities
  - Reflected
  - Refracted (transmitted)
  - Absorbed



25-6: Index of Refraction

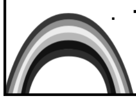
- **The wave incident on an interface can not only reflect, but it can also propagate into the second material.**
- **Claim the speed of an electromagnetic wave is different in matter than it is in vacuum.**
  - **Recall, from Maxwell's eqns in vacuum:** $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$
  - **How are Maxwell's eqns in matter different?**
    - **$\varepsilon_0 \rightarrow \varepsilon$, $\mu_0 \rightarrow \mu$ (both increase)**
    - **Therefore, the speed of light in matter is smaller and related to the speed of light in vacuum by:**

$$v = \frac{c}{n}$$

where n = index of refraction of the material: $n \approx \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{\kappa} > 1$

- **The index of refraction is frequency dependent: For example**

$n_{blue} > n_{red}$

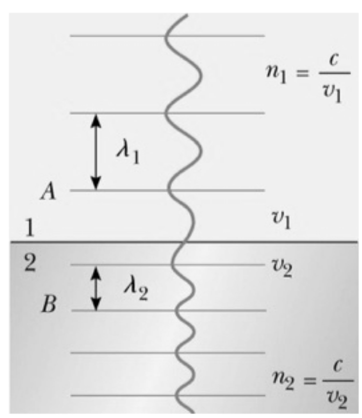


Frequency Between Media

- **As light travels from one medium to another, *its frequency does not change:***

$v = f\lambda$

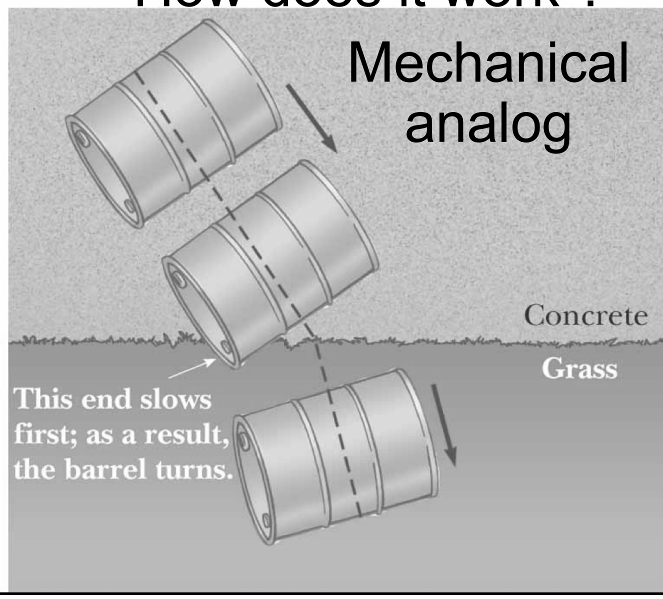
  - **Both the wave speed and the wavelength do change**
  - **The wavefronts do not pile up, nor are created or destroyed at the boundary, so *f* must stay the same**
- The ratio of the indices of refraction of the two media can be expressed as various ratios



$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

$\lambda_1 n_1 = \lambda_2 n_2$

How does it work ?

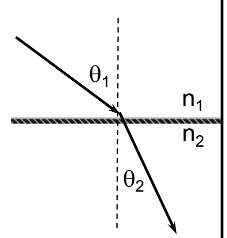


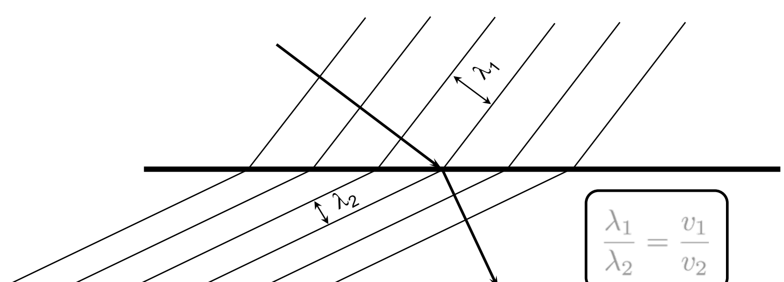
Refraction

- How is the angle of refraction related to the angle of incidence?
  - Unlike reflection, θ_1 cannot equal θ_2 !!
    - » $n_1 \neq n_2 \Rightarrow v_1 \neq v_2$

      but, the frequencies (f_1,f_2) must be the same $\Rightarrow$ the wavelengths must be different!

      Therefore, θ_2 must be different from θ_1 !!



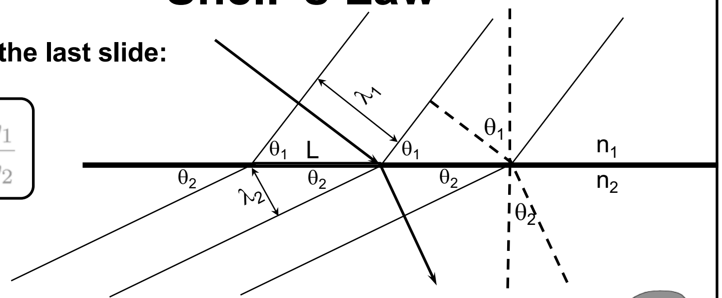


$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

Snell's Law

- **From the last slide:**

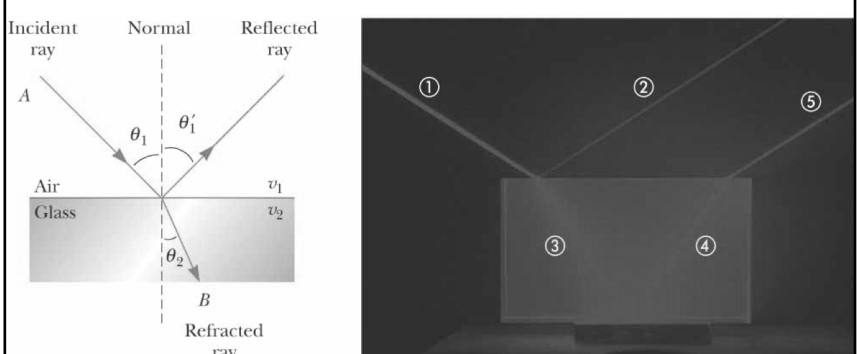
$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$



The two triangles above each have hypotenuse L

$$\therefore \quad L = \frac{\lambda_2}{\sin\theta_2} = \frac{\lambda_1}{\sin\theta_1} \quad \Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

But, $\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \quad \Rightarrow \quad n_1 \sin\theta_1 = n_2 \sin\theta_2$



(a) (b)

26-5 The Refraction of Light

- Here are some typical indices of refraction:

TABLE 26-2 Index of Refraction for Common Substances

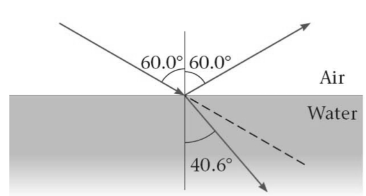
Substance	Index of refraction, n
SOLIDS	
Diamond	2.42
Flint glass	1.66
Crown glass	1.52
Fused quartz (glass)	1.46
Ice	1.31
LIQUIDS	
Benzene	1.50
Ethyl alcohol	1.36
Water	1.33
GASES	
Carbon dioxide	1.00045
Air	1.000293

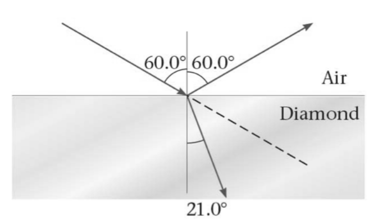
26-5 The Refraction of Light

- We can now write the angle of refraction in terms of the index of refraction:

Snell's Law

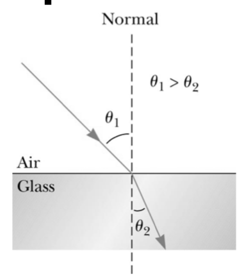
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

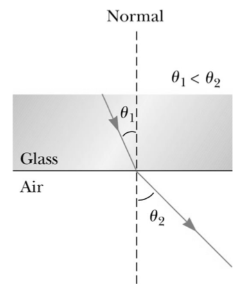




26-5 Refraction: Basic properties

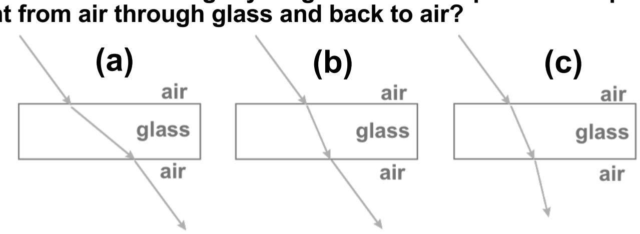
- Light may refract into a material where its speed is lower
- angle of refraction is less than the angle of incidence
  - The ray bends *toward* the normal
- Light may refract into a material where its speed is higher
- angle of refraction is more than the angle of incidence
  - The ray bends *away from* the normal
- If $n_1 = n_2 \Rightarrow$ no effect
- If light enters normal $\Rightarrow$ no effect





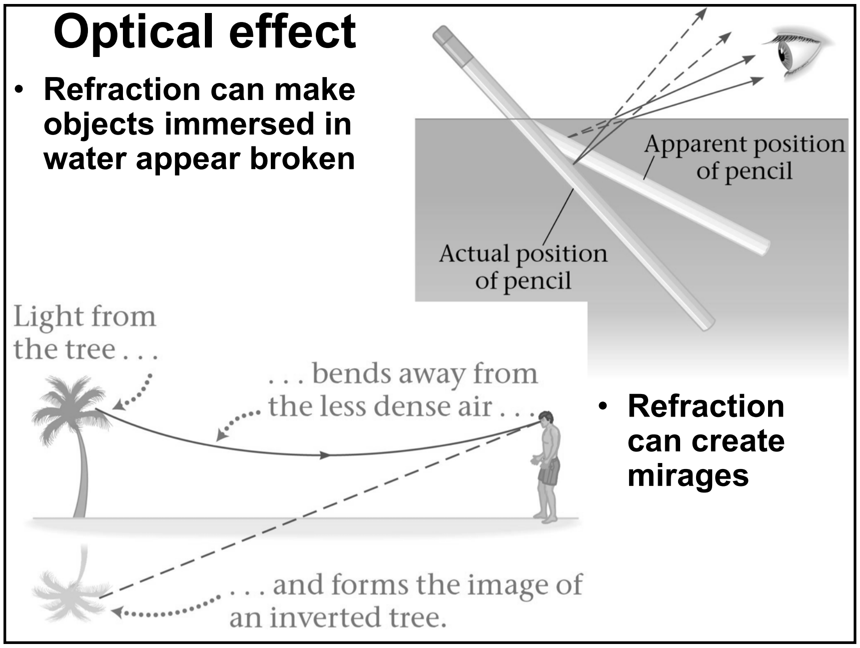
Lecture 17, ACT 4

- Which of the following ray diagrams could represent the passage of light from air through glass and back to air?



Optical effect

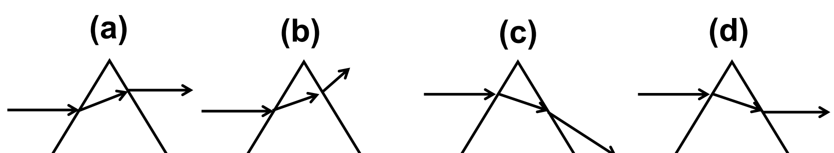
- Refraction can make objects immersed in water appear broken



- Refraction can create mirages

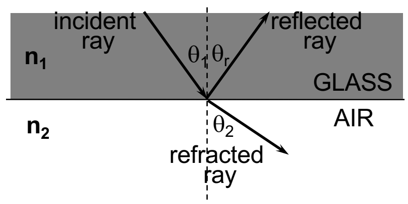
Lecture 17, ACT 5

- Which of the following ray diagrams could represent the passage of light from air through glass and back to air?



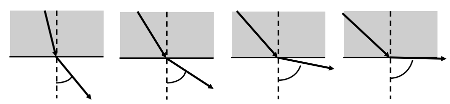
Total Internal Reflection

– **Consider light moving from glass (n_1=1.5) to air (n_2=1.0)**



$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2} > 1 \Rightarrow \theta_2 > \theta_1$$

ie light is bent away from the normal. as θ_1 gets bigger, θ_2 gets bigger, but θ_2 can never get bigger than 90° !!



In general, if $\sin\theta_1 \geq \sin\theta_C \geq (\mathbf{n_2} / \mathbf{n_1})$, we have NO refracted ray; we have TOTAL INTERNAL REFLECTION.

For example, light in water which is incident on an air surface with angle $\theta_1 > \theta_c = \sin^{-1}(1.0/1.5) = 41.8°$ will be totally reflected. This property is the basis for the optical fibers used in communication.

ACT 6: Critical Angle...

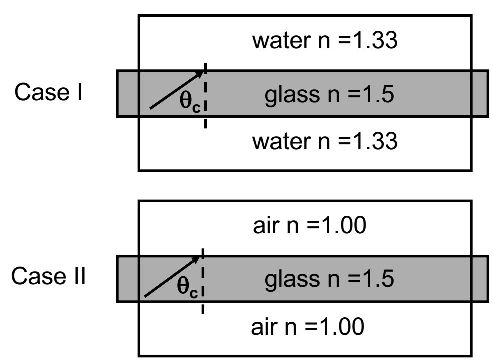
An optical fiber is cladded by another dielectric. In case I this is water, with an index of refraction of 1.33, while in case II this is air with an index of refraction of 1.00.

Compare the critical angles for total internal reflection in these two cases

a) $\theta_{cI} > \theta_{cII}$

b) $\theta_{cI} = \theta_{cII}$

c) $\theta_{cI} < \theta_{cII}$

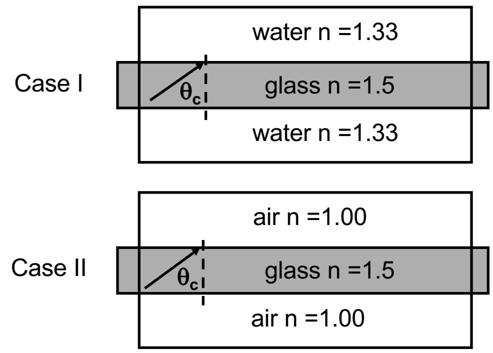


ACT 7: Fiber Optics

The same two fibers are used to transmit light from a laser in one room to an experiment in another. Which makes a better fiber, the one in water (I) or the one in air (II) ?

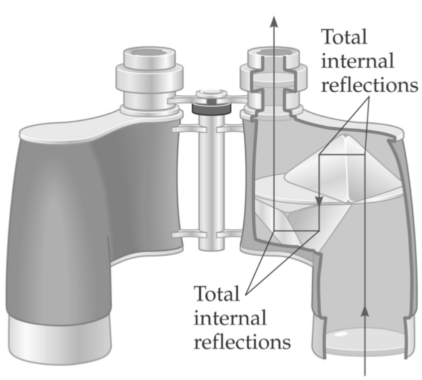
a) I Water

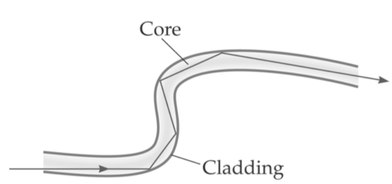
b) II Air



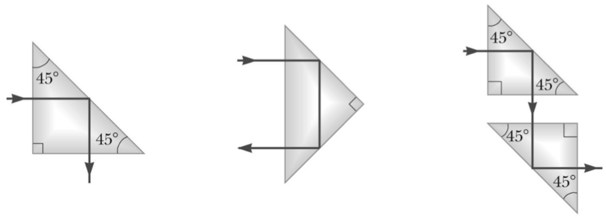
26-5 Applications of TIR

- **Total internal reflection (TIR) is used**
  - **in some binoculars**
  - **in optical fibers**
  - **In other optical devices**





Internal Reflection in a Prism



Submarine Periscopes to "see around corners"

Fiber Optics

- An application of internal reflection
- Plastic or glass rods are used to "pipe" light from one place to another

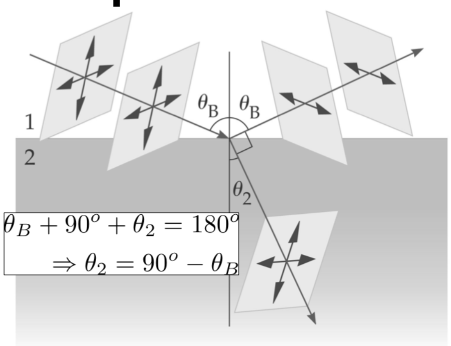
Dennis O'Clair/Tony Stone Images/Getty Images

Hank Morgan/Photo Researchers, Inc.

- Applications include
  - medical use of fiber optic cables for diagnosis and correction of medical problems
  - Telecommunications

26-5 Refraction & polarization

- **Brewster's angle**
  - **special angle**
  - **light reflected is totally polarized**
- **Reflected light is completely polarized**
  - **When angle between reflected and refracted beams is 90°**
  - **Polarization is parallel to the reflecting surface**
- **Applications**
  - **Remove reflection**
  - **Photographs of objects in water …**



$$\theta_B + 90^o + \theta_2 = 180^o$$
$$\Rightarrow \theta_2 = 90^o - \theta_B$$

$$n_1 \sin\theta_B = n_2 \sin(90^o - \theta_B)$$
$$= n_2 \cos\theta_B$$

$$\frac{n_2}{n_1} = \frac{\sin\theta_B}{\cos\theta_B} = \tan\theta_B$$

Recap of Today's Topic :

- **Announcements:** **Team problems today**
  - **Team 10, 11 & 12: This Thursday**
- **Homework #8: due Friday**
- **Midterm 2:**
  - **Tuesday April 10**
  - **Office hours if needed (M-2:30-3:30 or TH 3:00-4:00)**
- **Chapter 26:**
  - **Geometrical Optics**
  - **Mirrors equation**
  - **Refraction**