Physics 151 - Formula

Uniformly accelerated motion: \( <v> = (v + v_0)/2 \); \( x = v_0 t + 1/2 a t^2 \); \( v = v_0 + a t \); \( v^2 - v_0^2 = 2 a \Delta x \)

Newton laws of motion: 1. absence of net force, \( v \) = constant; 2. \( F = ma \); 3. \( F_{A,B} = -F_{B,A} \)

Forces, gravity \( W = mg \); Friction \( F_{\text{max}} = \mu_s N \) (static) \( F = \mu N \) (kinetic), spring \( F_s = -kx \)

Kinetic Energy: \( K = \frac{1}{2} m v^2 \) linear motion; \( K = \frac{1}{2} I \omega^2 \) rotational motion

Potential energy gravity: \( U = mgh \)

spring: \( U = 1/2 k x^2 \)

conservation of total energy: \( E = U + K \)

linear momentum: \( \mathbf{p} = m \mathbf{v} \) \( (\mathbf{F} = d\mathbf{p}/dt) \)

conservation of linear momentum if \( F_{\text{ext}} = 0 \) \( \rightarrow d\mathbf{p}/dt = 0 \) or \( \mathbf{p}_1 = \mathbf{p}_2 \)

Impulse \( = \int F(t) \, dt = \Delta \mathbf{p} \)

Momentum Conservation: \( \Sigma \mathbf{P}_i = \Sigma \mathbf{m} \mathbf{v}_i = \text{constant} \)

Elastic collision: \( \Sigma K_i = \Sigma \frac{1}{2} m v_i^2 = \text{constant} \)

Center of mass: \( M \mathbf{X}_{\text{cm}} = m_1 x_1 + m_2 x_2 + ... \)

Angular kinematics: \( \dot{\phi} = 1/2(\phi + \phi_0) ; \theta = \phi t ; \Delta \theta = \phi_0 t + 1/2 \omega t^2 ; \omega = \phi + \alpha t ; \omega^2 = \omega_0^2 + 2 \alpha \theta \)

Tangential quantities: \( S = r \theta ; v = r \omega ; a = r \alpha \)

Circular motion: \( a_{\text{cent}} = \omega^2 r = v^2/r \) also \( \omega = 2\pi R/T \)

Moment of inertia: \( I = \Sigma r_i^2 m_i \)

\( I = I_{\text{CM}} + MR^2 \) (parallel axis)

\( I = MR^2 \) for cylindrical shell about its CM or a hoop.

\( I = \frac{1}{2} MR^2 \) for cylinder or disk about its CM

\( I = 1/12 ML^2 \) for a thin rod about its CM

\( I = 2/5 MR^2 \) for a sphere about its CM

Rotational Kinetic Energy: \( K = 1/2 I \omega^2 \)

Combined rotation plus linear motion: \( K = 1/2 mv^2 + 1/2 I \omega^2 \)

Rolling without slipping: \( v_{\text{cm}} = R \omega \)

torque as a vectorial (cross) product: \( \tau = r \times \mathbf{F} ; \tau = I \alpha \)

angular momentum: \( \mathbf{L}_z = r x p = I \omega ; \tau = d\mathbf{L}/dt \)

Power \( P = \tau \omega \)

conservation of angular momentum \( (\tau_{\text{ext}} = 0) \) then \( d\mathbf{L}/dt = 0 ; \) or \( \mathbf{L}_1 = \mathbf{L}_2 \)

static equilibrium: \( \Sigma \mathbf{F}_i = 0 \) and \( \Sigma \tau_i = 0 \)

center of mass: \( R_{\text{cm}} = \Sigma r_i m_i / M_{\text{total}} \)

Mass on a spring \( \omega = \sqrt{k/m} \); \( T = 2\pi/\omega ; f = 1/T = \omega/2\pi ; \omega = 2\pi f \)

Displacement: \( x = x_0 \cos(\omega t + \phi) \)

Velocity: \( v = -v_0 \sin(\omega t + \phi) \) \( (v_0 = \omega x_0) \)

Acceleration: \( a = -\omega^2 x = -\omega^2 x_0 \cos(\omega t + \phi) \)

pendulum: \( T = 2\pi \sqrt{L/g} \) \( \omega = \sqrt{g/L} \); Resonance at \( \omega = \omega_0 \)

general pendulum \( \omega = \sqrt{mgd/I} \); Torsional Pendulum \( \omega = \sqrt{k/I} \)
Gravity
Newton's Law of Gravity: \( F = G\frac{m_1 m_2}{r^2} \)
\( g = \frac{GM_E}{R_E^2} \)
Kepler's Third Law: \( T^2 = K\beta r^3 \)
Second Law: \( \frac{dA}{dt} = \text{constant} \)
Gravitational Potential: \( U = -G\frac{m_1 m_2}{r} \)
Total Energy of Satellite \( E = G\frac{m_1 m_2}{2r} - G\frac{m_1 m_2}{r} = -G\frac{m_1 m_2}{2r} \)
escape velocity: \( v = \sqrt{\frac{2GM_E}{R_E}} \)

Fluids
Statics: Pressure: \( P = \frac{F}{A} = P_0 + \rho gh \) (P constant at a given height)
continuity equation: \( Av = \text{constant} \)
Dynamics: Bernoulli's equation: \( P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant.} \)
Archimede’s Principle: \( B = \rho_{\text{fluid}} V_{\text{disp}} \delta_g \)

Waves
In general,
Any Wave: \( y(x,t) = f(x-vt) \)
Harmonic Waves: \( y(x,t) = A\cos(kx - \omega t + \phi) \)
Superposition: \( y_{\text{total}} = y_1(x,t) + y_2(x,t) \) out of phase by \( \phi \)
\( y_{\text{total}} = 2A\cos(\phi/2)\cos(kx-\omega t+\phi/2) \)
Wave number: \( k = \frac{2\pi}{\lambda} \)
Angular frequency: \( \omega = 2\pi f = 2\pi/T \)
phase velocity: \( v = \omega/k = \lambda/T = \lambda f \)
velocity: \( v = \sqrt{\frac{E}{\mu}} \) string
power: \( \overline{P} = \frac{1}{2} \mu \omega^2 A^2 \) string; \( \overline{P} = \frac{1}{2} \rho (Area)\omega^2 A^2 \) sound
energy per unit length: \( \frac{dE}{dx} = \frac{1}{2} \mu \omega^2 A^2 \) Intensity: \( I = \frac{\overline{P}}{Area} = \frac{1}{2} \rho \omega^2 A^2 \)

Standing waves on a string: \( \lambda = 2L/n \)

Thermo
\( PV = nRT \) or \( PV = nKT \)
Conversion \( 0°C = 273 K \) unit size the same, or \( K = °C + 273; \) \( °C = (°F - 32)\times\frac{5}{9} \)
\( \Delta L = \alpha L \Delta T; \) \( \Delta V = \beta V \Delta T; \beta = 3\alpha \)
\( \alpha \) for some materials, all times \( 10^6 \): Al 24; Brass 19; Cu 17, glass 9, Pyrex 3.2, lead 29, steel 11, concrete 12, Invar 0.9
\( \beta \) for some materials, all times \( 10^{-4} \): Alcohol 1.12, benzene 1.24, acetone 1.5, glycerin 4.85, Mercury 1.82, turpentine 9.0, gasoline 9.6, air 36.7, Helium 36.7
\( Q = mc\Delta T \) defines \( c \), specific heat
\( Q = mL \) defines \( L \), latent heat
\( \Delta E_{\text{int}} = Q + W \)