Lecture 25

Griffiths, section 12.2.4 4/25/23

3D-force in the Relativistic Dynamics.

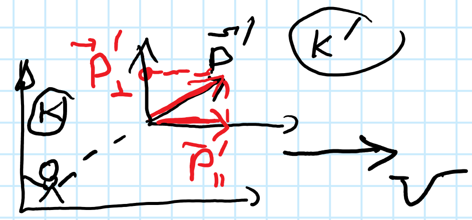
The definition of the force $\vec{F}$ is the same in the classical and relativistic mechanics $\Rightarrow$ $\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$

$$\boxed{\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}}$$

$\vec{F}$ is not a relativistic invariant. (in the classical mechanics $\vec{F}$ is an invariant). $\vec{v} = \frac{d\vec{r}}{dt}$

Transformation of the force projections $\vec{F}_\perp$ and $\vec{F}_\parallel$

$\vec{p} = \vec{p}_\parallel + \vec{p}_\perp$ $\quad (\vec{p}_\parallel \parallel \vec{V},\ \vec{p}_\perp \perp \vec{V})$



The Lorentz transformations:

$$\vec{p}_\perp' = \vec{p}_\perp \quad \text{and} \quad d\vec{p}_\perp' = d\vec{p}_\perp$$

Lab Frame

$$\vec{F}_\perp = \frac{d\vec{p}_\perp}{dt} = \frac{dp_\perp'}{dt}$$

$$dt = \frac{dt' + \frac{V}{c^2}dx'}{\sqrt{1 - V^2/c^2}} = dt' \frac{1 + \frac{\vec{v}'\cdot\vec{V}}{c^2}}{\sqrt{1 - V^2/c^2}}$$

$$\vec{F}_\perp = \frac{d\vec{p}_\perp' \cdot \sqrt{1 - V^2/c^2}}{dt'\left(1 + \frac{\vec{v}'\cdot\vec{V}}{c^2}\right)} = \vec{F}_\perp' \cdot \frac{\sqrt{1 - V^2/c^2}}{1 + \frac{\vec{v}'\cdot\vec{V}}{c^2}}$$

$d\vec{p}_\parallel'/dt$

$$\vec{F}_\parallel = \frac{d\vec{p}_\parallel}{dt} = \frac{(dp_\parallel' + \frac{V}{c^2}d\varepsilon')\cdot\cancel{\sqrt{1 - V^2/c^2}}}{\cancel{\sqrt{1 - \frac{V^2}{c^2}}}\cdot dt'\left(1 + \frac{\vec{v}'\vec{V}}{c^2}\right)} = \frac{\vec{F}_\parallel' + \frac{V}{c^2}\frac{d\varepsilon'}{dt'}}{1 + \frac{\vec{v}'\vec{V}}{c^2}}$$

$$\frac{d\varepsilon'}{dt'} = \frac{d\vec{p}'}{dt'}\frac{\partial\varepsilon'}{\partial\vec{p}'} = \vec{F}'\cdot\vec{v}'$$

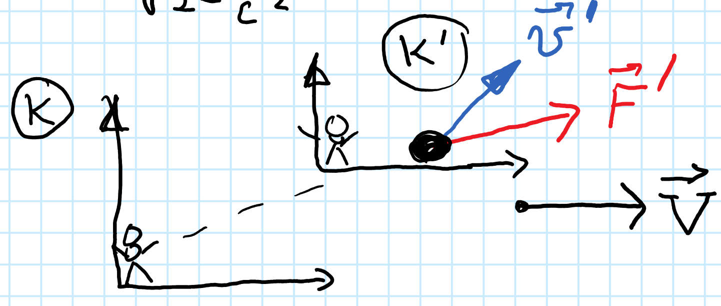
$\vec{F}'$

because $\frac{\partial\varepsilon}{\partial\vec{p}} = \frac{\partial}{\partial p}\left(\sqrt{p^2c^2 + m^2c^4}\right) = c^2\frac{\cancel{2}\vec{p}}{\cancel{2}\varepsilon(p)} = \vec{v}$

$\varepsilon(p) = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} = \vec{v}\,\frac{\varepsilon(p)}{c^2}$$

$$\boxed{\begin{aligned} \vec{F}_\parallel &= \frac{\vec{F}_\parallel' + \frac{V}{c^2}(\vec{v}'\cdot\vec{F}')}{1 + \frac{\vec{v}'\cdot\vec{V}}{c^2}} \\ \vec{F}_\perp &= \vec{F}_\perp' \cdot \frac{\sqrt{1 - V^2/c^2}}{1 + \frac{\vec{v}'\cdot\vec{V}}{c^2}} \end{aligned}}$$



Symplest case: K' is the proper frame of the moving object. $\Rightarrow \vec{v}' = 0$

$$\boxed{\vec{F}_\parallel = \vec{F}_\parallel' \quad \text{and} \quad \vec{F}_\perp = \vec{F}_\perp' \cdot \sqrt{1 - \frac{V^2}{c^2}}}$$