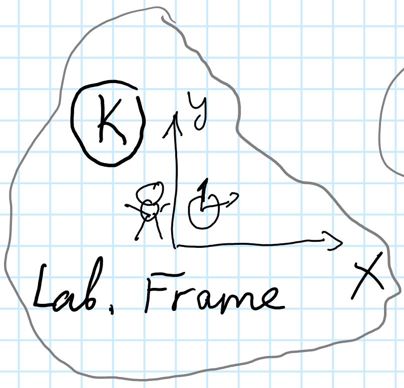
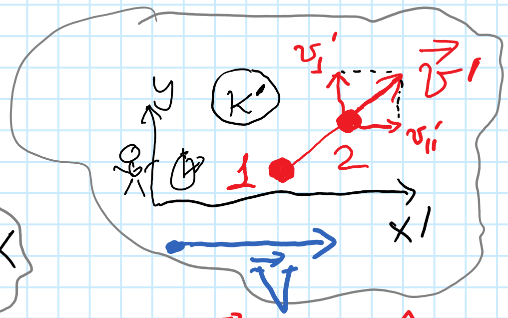
Lecture 17 03/28/2023

recapitulation: Transformation of Velocities





$\vec{v}_{\parallel} = v_x' \cdot \hat{e}_x$

$\boxed{\text{K- frame velocity}}$

Particle velocities:

$\hat{e}_x = \hat{e}_x' \; ; \; \hat{e}_y = \hat{e}_y' \; ; \; \hat{e}_z = \hat{e}_z'$

$$\vec{V} = \hat{e}_x V$$

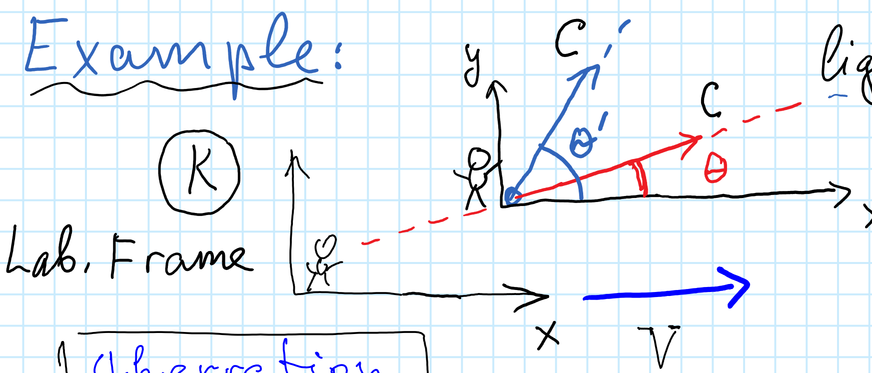
The same formulas expressed in the vector form:

$$\boxed{\vec{v}_{\parallel} = \frac{\vec{v}_{\parallel}' + \vec{V}}{1 + \frac{\vec{v}'\vec{V}}{c^2}}} \qquad \boxed{\vec{v}_{\perp} = \vec{v}_{\perp}' \cdot \frac{\sqrt{1 - V^2/c^2}}{1 + \frac{\vec{v}' \cdot \vec{V}}{c^2}}}$$

$\vec{v}_{\parallel}' \parallel \vec{V}$

$\vec{v}_{\perp}' \perp \vec{V}$

Example: light propagation in K and K' frames



$\boxed{\text{Aberration of light}}$

$v = v' = c$

$$\boxed{\cos\theta = \frac{\cos\theta' + \frac{V}{c}}{1 + \frac{V}{c}\cos\theta'}} \qquad \theta' \to \theta$$

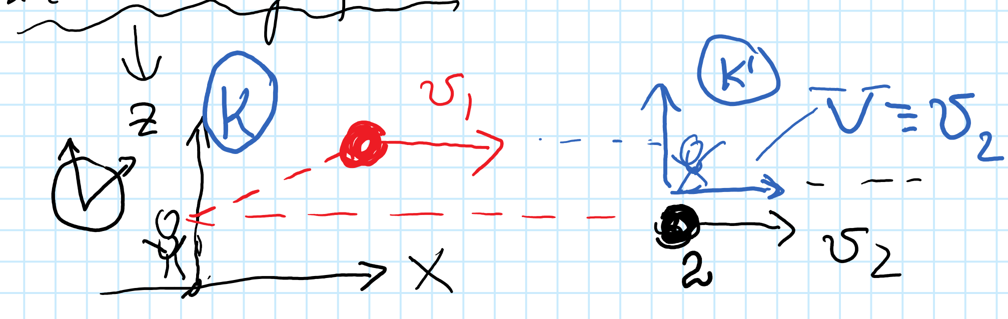
$(v_x' = c \cdot \cos\theta' \; ; \; v_y' = c \cdot \sin\theta')$

Transformation $\theta \to \theta'$:

$$V \to -V \qquad \boxed{\cos\theta' = \frac{\cos\theta - \frac{V}{c}}{1 - \frac{V}{c}\cos\theta}}$$

Relative velocity (example)

Laboratory frame



two articles with velocities v_1 and v_2.

Calculate the relative velocity of particle "1" with respect to the particl "2".

In the relativistic case: this is the velocity of particle "1" in the frame K' moving with the velocity v_2.

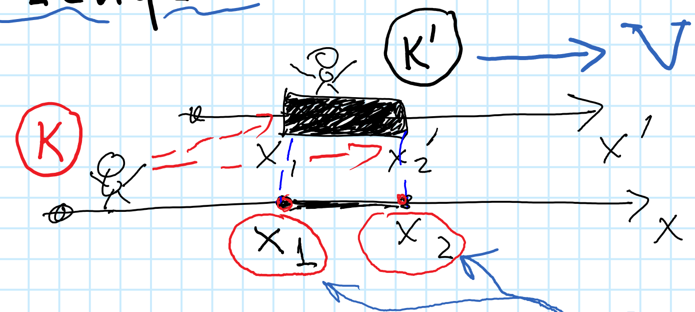
K': $V = v_2 \Rightarrow$

The realive velocity $\vec{v}_r$:

$$\boxed{v_r \equiv v_1' = \frac{v_1 - V}{1 - \frac{v_1 V}{c^2}} = \frac{v_1 - v_2}{1 - \frac{v_1 v_2}{c^2}}}$$

$$v' = \frac{v - V}{1 - \frac{vV}{c^2}}$$

Proper Length



In the K-frame:
There are two events: measurements x_1 and x_2.

and it has to be done simultaneously at $t_1 = t_2$:

$$x_1 = \frac{x_1' + V t_1'}{\sqrt{1-\frac{V^2}{c^2}}} \qquad x_2 = \frac{x_2' + V t_2'}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$\boxed{t_1 = \frac{t_1' + \frac{V}{c^2} x_1'}{\sqrt{1 - V^2/c^2}} \qquad t_2 = \frac{t_2' + \frac{V}{c^2} x_2'}{\sqrt{1-\frac{V^2}{c^2}}}}$$

$\Rightarrow t_1 = t_2$

$$t_2' - t_1' = \frac{V}{c^2}(x_1' - x_2')$$

$$\boxed{x_2' - x_1' \equiv \ell_0}$$

rod length (proper length)

$$x_2 - x_1 = \frac{x_2' - x_1' + V(t_2' - t_1')}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{(x_2' - x_1')\left(1 - \frac{V^2}{c^2}\right)}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$x_2 - x_1 = (x_2' - x_1')\sqrt{1-\frac{V^2}{c^2}} = \ell_0\sqrt{1-\frac{V^2}{c^2}}$$

Lorentz contraction:

$$\boxed{\ell = \ell_0 \sqrt{1 - \frac{V^2}{c^2}}}$$

proper length

Another method

$$t_1 = t_2 \begin{cases} x_1' = \dfrac{x_1 - V t_1}{\sqrt{1-\frac{V^2}{c^2}}} \\ x_2' = \dfrac{x_2 - V t_2}{\sqrt{1-\frac{V^2}{c^2}}} \end{cases} \Leftarrow \begin{bmatrix} V \to -V \\ x \to x' \\ t \leftrightarrow t' \end{bmatrix}$$

$t_1 = t_2$

$$(x_2' - x_1') = \frac{x_2 - x_1}{\sqrt{1-\frac{V^2}{c^2}}} \Rightarrow \ell_0 = \frac{\ell}{\sqrt{1-\frac{V^2}{c^2}}}$$

Definition!

Lab. frame: $x_2 - x_1 = \ell$

"Proper" frame: $x_2' - x_1' = \ell_0$

Lab. Frame!

$$\ell = \ell_0 \cdot \sqrt{1 - \frac{V^2}{c^2}} \quad \leftarrow \text{length}$$

proper

$$\Delta t = \frac{\Delta \tau}{\sqrt{1-\frac{V^2}{c^2}}} \quad \leftarrow \text{time}$$

! $\boxed{\ell \cdot \Delta t = \ell_0 \cdot \Delta\tau}$

The phase volume of the 4-D space is the relativistic invariant:

$$\boxed{d^4x = d\tau\, dx' dy' dz' = dt\, dx\, dy\, dz}$$

$$d^4x = d^3r \cdot dt \equiv dx\, dy\, dz \cdot dt \equiv dx' dy' dz' dt'$$

The phase volume d^4x