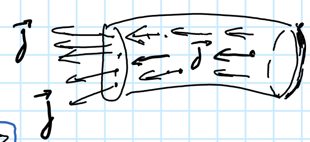
Lecture 2 — 01/19/23

Electric Current and Ohm's Law

Griffiths 5.1.3, 7.1.1

In previous lecture 1: $\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0}$ continuity equation (charge conservation)

Electric current I is a flux of charged particles crossing selected surface S. Relation between current I and $\vec{j}$:



$\boxed{\vec{j} = \rho\vec{v}}$

ρ – the local charge density
v – the local velocity of charge particles

$$\boxed{I = \int_S \vec{j}\cdot d\vec{S}}$$

Local velocity $\vec{v}$ depends on the local force $\vec{f}$.
The simplest dependence $\vec{v}(\vec{f})$ is → $\boxed{\vec{v} = b\cdot\vec{f}}$.
"b" is a constant defined as a mobility.
It depends on parameters of the medium conducting this electric current.

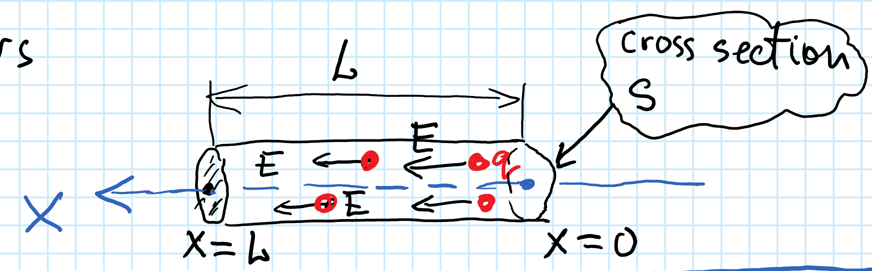
$$\boxed{\vec{j} = \rho\vec{v} = \rho b \vec{f}} \qquad \text{Eq. } (*)$$

The local force $\vec{f}$ can be a sum of several forces of different nature (electric, magnetic or gravitational).
For pure electric force, Eq. $(*)$ represents an experimental law:

Ohm's Law $\boxed{\vec{j} = \sigma\cdot\vec{E}}$ where $\boxed{\sigma = \rho\cdot b}$ is the conductivity.

$\begin{cases} \sigma \to \infty & \text{perfect conductors} \\ \sigma \to 0 & \text{perfect insulators} \end{cases}$

Example: cylindrical wire



$\varphi(L) - \varphi(0) = ?$

$$U = |\varphi(L) - \varphi(0)| = \int_0^L E\,dx = \frac{j}{\sigma}L = \frac{L}{S\sigma}\cdot I \Longrightarrow \boxed{U = I\cdot\left(\frac{L}{S\sigma}\right)}$$

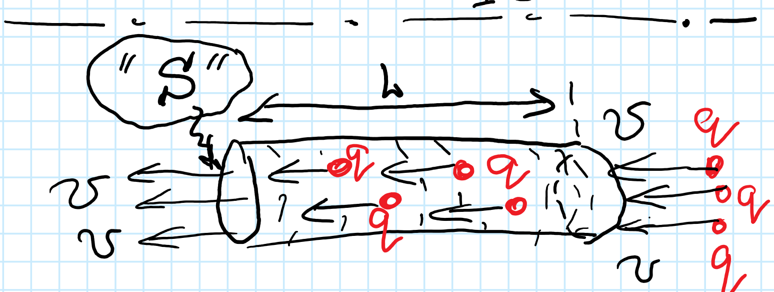
$|\vec{E}| = j/\sigma$ $\qquad$ $j = I/S$

Canonical form of Ohm's Law $\boxed{U = I\cdot R}$

$\boxed{R = \frac{L}{S\cdot\sigma}}$ ← Resistance

Units: $[R] = [\frac{V}{I}] = \text{Ohm}$

Joule Heating Law:



Work per particle: qU
Number of particles crossing "S"-surface per second: I/q

Power P = the total work per second

Power → $\boxed{P = \cancel{q}U\cdot\left(\frac{I}{\cancel{q}}\right) = U\cdot I}$

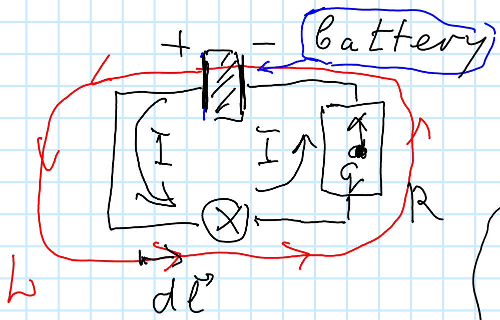
The volume power density:

$$P_v = \frac{P}{V} = \frac{U\cdot I}{L\,S} = \frac{U}{L}\cdot\frac{I}{S} = E\cdot j$$

$$\Longrightarrow \boxed{P_v = \sigma\cdot E^2}$$

Electromotive Force (emf)

Griffiths 7.1



$$\vec{f} = \vec{f}_s + \vec{E}$$

force per unit of charge

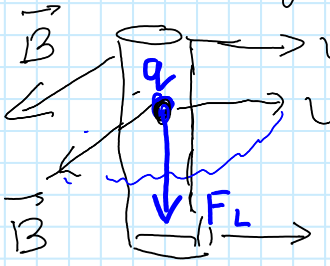
electrostatic force

$$\oint \vec{f}\, d\vec{l} = \oint \vec{f}_s\, d\vec{l} + \cancel{\oint \vec{E}\, d\vec{l}}^{\,0} = \mathcal{E}_s$$

This is the work → Electromotive force $\boxed{\mathcal{E}_s = \oint \vec{f}_s\, d\vec{l}}$ is a major characteristics of sources of electric power

Example: **Motional emf**

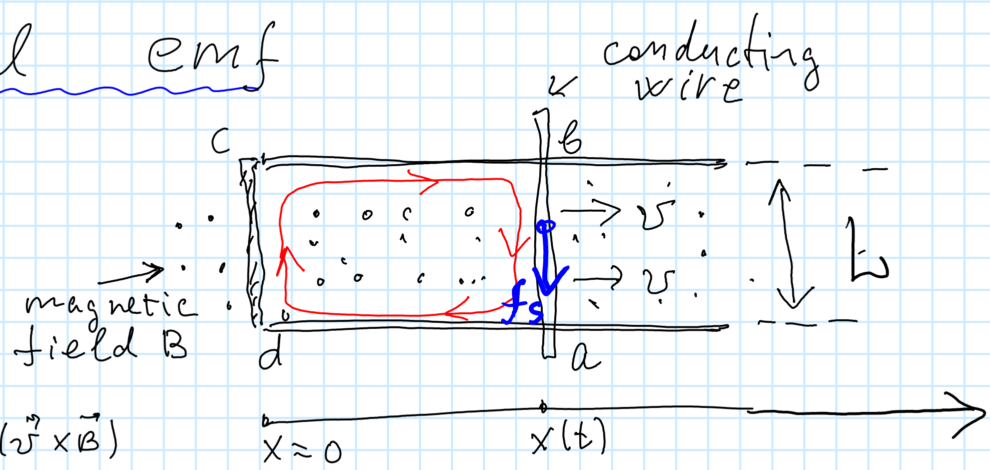
In moving conductor:



$$\boxed{\vec{f}_s = (\vec{v} \times \vec{B})}$$

$$\vec{f}_s = \frac{\vec{F}_L}{q} \qquad \vec{F}_L = q(\vec{v} \times \vec{B})$$

The Lorentz force

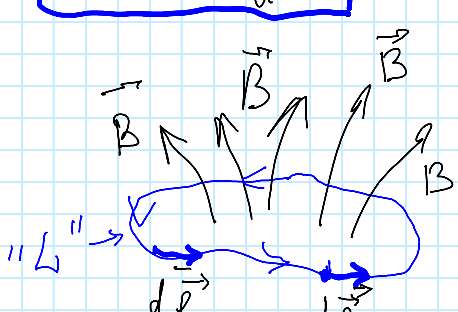


Electromotive force: $\mathcal{E}_s = \int_a^b \vec{f}_s\, d\vec{l} = -Lv \cdot B = -\frac{d}{dt}(SB) = -\frac{d\Phi}{dt}$

where $S = S(t) = L\,x(t) = Lvt$ – area of the current loop „abcd"

$\Phi = \Phi(t) = B \cdot S(t)$ – the magnetic flux through the "abcd" loop

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$



Faraday's Law (Electromagnetic Induction)

"Experimental Law" — variation of $\vec{B}$-field induce $\vec{E}$-field.

$$\boxed{\oint_L \vec{E}\, d\vec{l} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{S}}$$

$$\oint_L \vec{E}\, d\vec{l} \underset{\text{Stokes' Theorem}}{=} \int \vec{\nabla} \times \vec{E}\, d\vec{S} = -\int \frac{\partial \vec{B}}{\partial t} d\vec{S} \Longrightarrow \int d\vec{S}\,\underbrace{\left(\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t}\right)}_{0} = 0$$

Faraday's Law

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \leftarrow \text{differential form}$$

$$\boxed{\oint_L \vec{E}\, d\vec{l} = -\frac{\partial \Phi}{\partial t}} \leftarrow \text{integral form}$$

$$\Phi = \int_S \vec{B} \cdot d\vec{S}$$