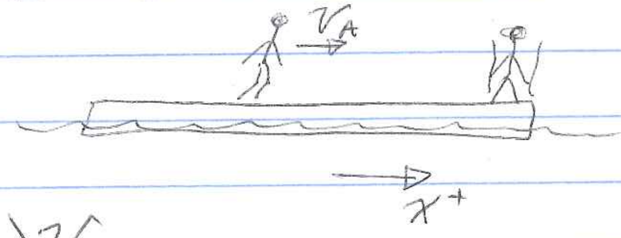
1. (a) Momentum is conserved



$P_i = 0$

$P_f = m_A v_A - (m_c + 2m_r + m_b) v_B$

$$P_f = P_i \implies v_B = \frac{m_A v_A}{m_c + 2m_r + m_b}$$

$$= \frac{(50\,kg)(1\,m/s)}{70\,kg + 2\times 1\,kg + 10\,kg}$$

$$= 0.6097561\ \frac{m}{s}$$

(b) No external forces - COM velocity conserved
COM starts at rest, stays at rest,
so COM position is constant

Measured from Alice's starting position:

$$M_{tot} x_{com} = \cancel{m_A \cdot 0} + m_b \cdot \frac{L}{2} + (m_c + 2m_r) L$$

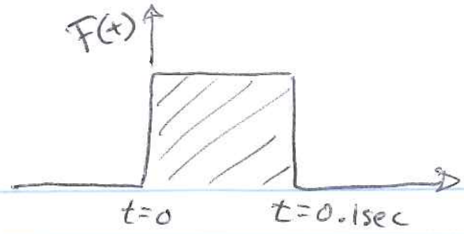
$$= (m_A + m_r) d + m_b \left(\frac{L}{2} + d\right) + (m_c + m_r)(L + d)$$

$$m_r L = (m_A + m_r) d + m_b d + (m_c + m_r) d$$

$$d = \frac{m_r L}{m_A + 2m_r + m_b + m_c}$$

$$= \frac{1\,kg\,(5\,m)}{50\,kg + 2\times 1\,kg + 10\,kg + 70\,kg} = 0.037878\,m$$

1 (c) Impulse to the right



$$I_{fish} = \Delta P = \int_{-\infty}^{\infty} F(t)\,dt = \int_0^{0.1sec} 100N\,dt = 10\ N\cdot sec = 10\ \frac{kg\,m}{s}$$

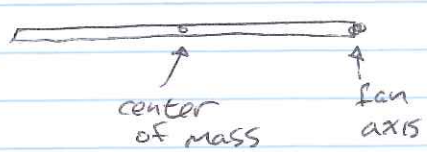
Now the total momentum is $I = 10\ kgm/s$ to the right

$$I_{fish} = (m_c + m_r)V_c - (m_A + m_r + m_b)V$$

$$V = \frac{I_{fish} - (m_c + m_r)V_c}{m_A + m_r + m_b}$$

$$= \frac{10\ kgm/s - (71kg)(1\ m/s)}{50\ kg + 1\ kg + 10\ kg} = 1\ m/s \text{ to the left}$$

2. (a)



parallel axis theorem:

$$I_{blade}^{(fan\ axis)} = I_{blade}^{(com)} + M\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12}ML^2 + \frac{1}{4}ML^2$$

$$= \frac{1}{3}ML^2$$

$$= 0.0333\ kg\ m^2$$

Four blades make the fan, so

$$I_{fan}^{(fan\ axis)} = 4 \times I_{blade}^{(fan\ axis)} = \frac{4}{3}ML^2 = \frac{4}{3}(0.1kg)(1m)^2$$

$$= 0.1333\ kgm^2$$

2. (b) $I^{(\text{fan axis})}_{\text{parakeet}} = M_p L^2 = 0.04\,\text{kg} \cdot (1\,\text{m})^2 = 0.04\,\text{kg}\,\text{m}^2$

(c) Angular momentum is conserved

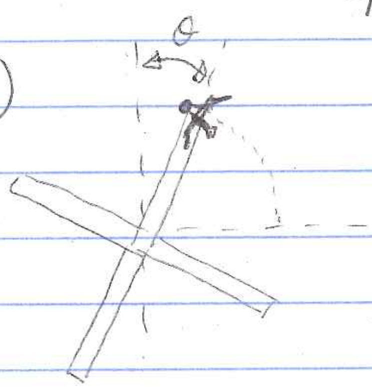
$L_i = 0$

$$L_f = I^{(\text{fan axis})}_{\text{parakeet}} \underbrace{\omega_{\text{parakeet}}}_{v/L} - I^{(\text{fan axis})}_{\text{fan}} \omega_{\text{fan}}$$

$$\omega_{\text{fan}} = \omega_{\text{parakeet}} \frac{I_{\text{parakeet}}}{I_{\text{fan}}} = \frac{v}{L} \frac{M_p \cancel{L^2}}{\frac{4}{3} M \cancel{L^2}}$$

$$= \frac{3v}{4L} \frac{M_p}{M} = \frac{3\,(1\,\frac{\text{m}}{\text{s}})}{4\,(1\,\text{m})} \cdot \frac{(0.04\,\text{kg})}{(0.1\,\text{kg})} = 0.30\,\frac{\text{rad}}{\text{sec}}$$

(d)



$$\theta = \omega_{\text{fan}} \cdot t = \frac{\pi}{2} - \omega_{\text{parakeet}} \cdot t$$

$$t = \frac{\pi}{2(\omega_{\text{fan}} + \omega_{\text{para}})}$$

$$\theta = \omega_{\text{fan}}\, t = \frac{\omega_{\text{fan}}\, \pi}{2(\omega_{\text{fan}} + \omega_{\text{para}})} = \frac{0.3\,\frac{\text{rad}}{\text{sec}} \cdot \pi}{2(0.3\,\frac{\text{rad}}{\text{sec}} + 1\,\frac{\text{rad}}{\text{sec}})}$$

$$= 0.3625\,\text{rad}$$

$$= 20.7692^\circ$$

3 (a) $$W = \int_0^d F \cdot dx = -\frac{1}{2}kd^2 = -\frac{1}{2}(100\ \text{N/m})(1\text{m})^2 = -50\ \text{J}$$

(b) Energy is conserved

$$E = \frac{1}{2}kd^2 = \frac{1}{2}(M+4m)v^2 + \frac{1}{2}(4I_{cyl})\omega^2$$

No slipping, $v = r\omega$

$$I_{cyl} = \frac{1}{2}mr^2$$

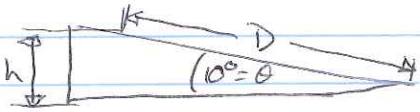
$$E = \frac{1}{2}kd^2 = \frac{1}{2}(M+4m)v^2 + \frac{1}{2}(4)\left(\frac{1}{2}m\cancel{r^2}\right)\left(\frac{v^2}{\cancel{r^2}}\right)$$

$$= \frac{1}{2}(M+6m)v^2$$

$$v = \sqrt{\frac{2(\frac{1}{2}kd^2)}{M+6m}} = \sqrt{\frac{2(50\text{J})}{10\text{ kg} + 6 \cdot 2\text{kg}}}$$

$$= \sqrt{\frac{50}{11}}\ \text{m/s} = 2.13201\ \text{m/s}$$

(c) $$E = \frac{1}{2}kd^2 = (M+4m)gh$$



$$= (M+4m)g\,D\sin\theta$$

$$D = \frac{(\frac{1}{2}kd^2)}{(M+4m)g\sin\theta} = \frac{50\text{ J}}{(10\text{kg} + 4 \times 2\text{kg})(9.81\text{ m/s}^2)\sin 10^\circ}$$

$$= 1.63064\ \text{m}$$

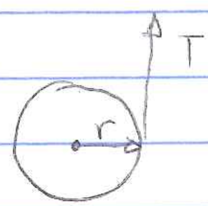
4. (a) $I_{total} = 2 \times \frac{1}{2} M R^2 + \frac{1}{2} m r^2$

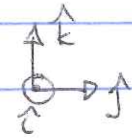
$$= M R^2 + \frac{1}{2} m r^2$$

$$= (0.05 kg)(0.03 m)^2 + \frac{1}{2}(0.005 kg)(0.01 m)^2$$

$$= 4.525 \times 10^{-5} \, kg\, m^2$$

(b)





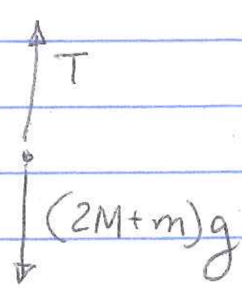
$\vec{\tau} = \vec{r} \times \vec{F} = r T \,\hat{\imath} = I \vec{\alpha}$

Torque and angular acceleration vectors $\parallel \hat{\imath}$

(c) Acceleration of the yo-yo along $-\hat{k}$ causes a positive angular acceleration along $\hat{\imath}$ (right-hand rule)

$$a_z = -r\alpha$$

(d)



$$(2M+m) a_z = -(2M+m) g + T$$

$$r T = I \alpha = I\left(-\frac{a_z}{r}\right)$$

$$T = -\frac{I a_z}{r^2} = (2M+m)(a_z + g)$$

$$-\frac{I a_z}{r^2} - (2M+m) a_z = (2M+m) g$$

$$a_z = -\frac{(2M+m) g}{\frac{I}{r^2} + 2M + m} = -1.84762 \, m/s^2$$