Midterm #2 Solutions

Physics 1501Q, Sections 21L-26L, Fall 2013

There are often many ways to arrive at an answer. For some conceptual problems, it happens through the intuition we have gained since birth about how the natural world works. For problems which use mathematics, there are often multiple approaches that works equally well. This solution set is only one possible account of reasoning through the test.

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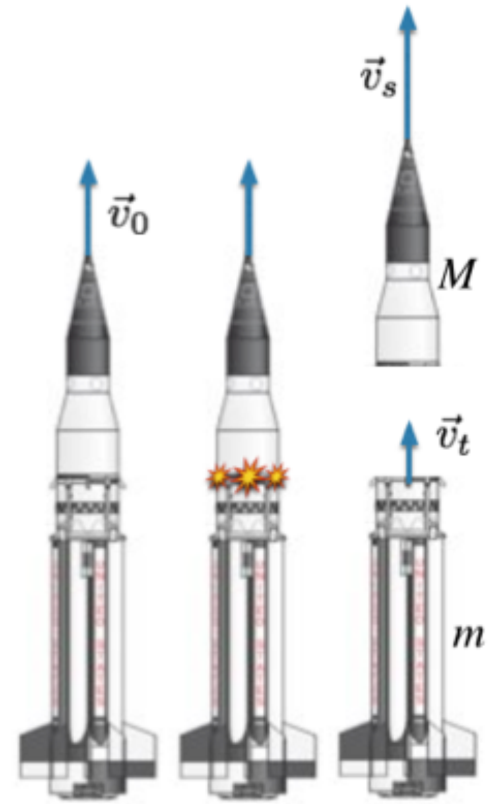
1. The final stage of a space shuttle (M=2 million kg) launch sequence involves the release of the empty external tank (m=350,000 kg). With all rockets off, the shuttle and tank move at a constant speed v_0=7950m/s vertically. A small explosion separates the tanks and shuttle, pushing the tank away directly behind the shuttle. Neglect effects of gravity.



(a) If the shuttle is now moving v_s=8000m/s vertically, what is the tank's speed v_t?

We can neglect gravity, so there are no external forces and momentum is conserved. Momentum of the shuttle plus tank is the same before and after the explosion.

$$(M+m)v_0 = Mv_s + mv_t$$

$$v_t = \frac{(M+m)v_0 - Mv_s}{m}$$

(b) The explosion lasts 1.5 seconds. What was the average force on the shuttle during the explosion?

The impulse felt by the rocket is the change in its momentum as a result of the explosion. This is related to the average force and known duration of the event:

$$I = Mv_s - Mv_0 = F_{avg}\Delta t$$

$$F_{avg} = \frac{Mv_s - Mv_0}{\Delta t}$$

2. A hoop ($I_{hoop,cm}=mR^2$) and a sphere ($I_{sphere,cm}=2/5mR^2$) with the same mass and radius roll without slipping along a flat surface with the same speed V. They each then roll up a ramp to their maximum heights H_h *and* H_s before stopping to return down the ramp. Determine the ratio H_h/H_s.

All forces are conservative (no friction), so we can use energy to relate the change in kinetic energy to the change in potential energy. Each object has the same amount of kinetic energy in its translational motion, but these shapes roll without slipping, though and we must consider the kinetic energy associated with the rotation as well.

Energy conservation: $$E = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 = mgH$$

No-slip condition: $$V = R\omega$$

The one with the higher MOI (the hoop) has a larger rotational kinetic energy and so a higher total kinetic energy. This KE is going to be converted into PE, so the hoop will go higher up the ramp.

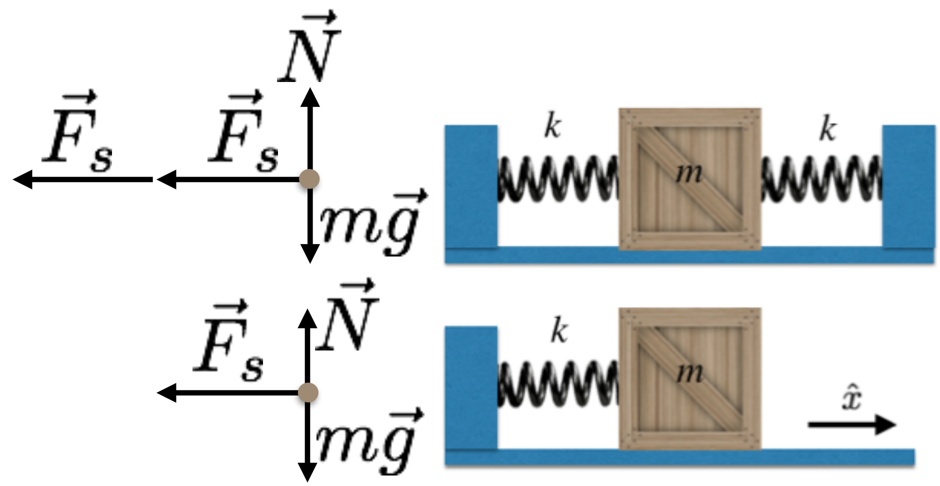
$$H_h = (1 + \frac{I_h}{mR^2})\frac{V^2}{2g} = 2\frac{V^2}{2g}$$

$$H_s = (1 + \frac{I_s}{mR^2})\frac{V^2}{2g} = \frac{7}{5}\frac{V^2}{2g}$$

$$\frac{H_h}{H_s} = \frac{10}{7}$$

3. Consider springs (spring constant k=18N/m) attached to a box (mass m=0.5kg) above a frictionless surface in the two arrangements shown. The springs are at their equilibrium length in each case.

(a) Draw a free body diagram for each case when the box is displaced to the right.



(b) Which case has the higher oscillation frequency? Why?

For motion in the horizontal direction, we have from the FBDs

Two springs: $$m\frac{d^2x}{dt^2} = -2kx \qquad \frac{d^2x}{dt^2} = -2\frac{k}{m}x \qquad \omega_2 = \sqrt{\frac{2k}{m}}$$

One spring: $$m\frac{d^2x}{dt^2} = -kx \qquad \frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad \omega_1 = \sqrt{\frac{k}{m}}$$

The box with two springs will oscillate at a frequency that is $\sqrt{2}$ times higher than the box with one spring

(c) The box in the lower panel starts from equilibrium and is kicked at time t=0, so that it moves to the left initially at a speed 1m/s. Find the position $x(t)$ for the box in terms of the time in seconds and plug in the numbers given.

The relevant equation governing the box position is:

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

with general solution

$$x(t) = A\cos(\omega t + \phi)$$

provided that

$$\omega = \sqrt{\frac{k}{m}} = 6\frac{rad}{sec}$$

We can determine A and phi by the initial conditions:

$$v(t) = \frac{dx(t)}{dt} = -A\omega\sin(\omega t + \phi)$$

$$v(0) = -A\omega\sin\phi = -1\frac{m}{s}$$

$$x(0) = A\cos\phi = 0$$

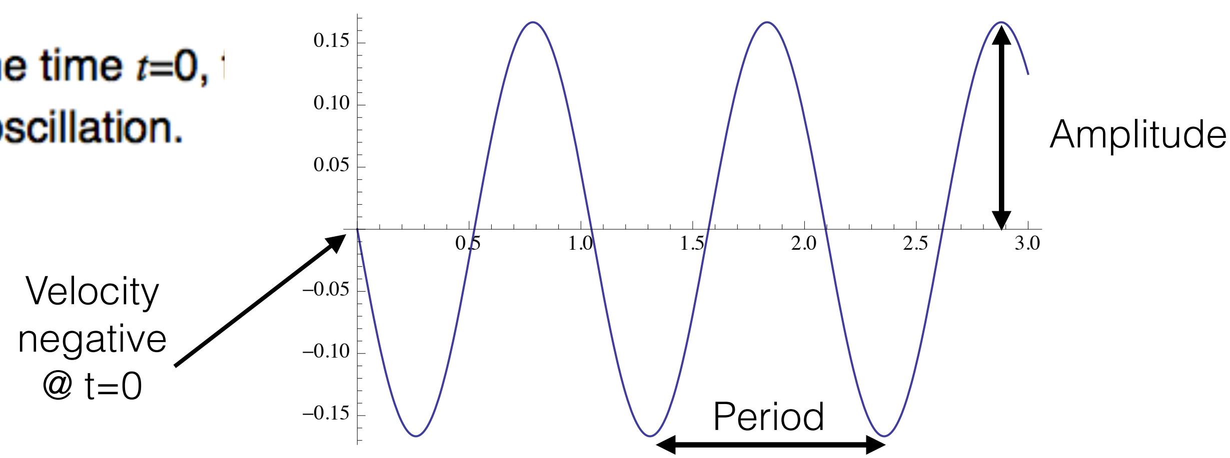
These can be true if

$$\phi = \frac{\pi}{2} \qquad A = \frac{1}{6}m$$

$$x(t) = (\frac{1}{6}m)\cos(6\frac{rad}{sec}t + \frac{\pi}{2})$$

$$x(t) = -(\frac{1}{6}m)\sin(6\frac{rad}{sec}t)$$

Sketch a graph of $x(t)$, labeling the time t=0,
the amplitude and period of the oscillation.



4. A fly fishing reel consists of a large cylinder ($I_{cyl,cm}=1/2MR^2$) with radius R=6cm and mass M=100g, with a small cylindrical handle of mass m=20g and radius r =1cm ($I_{handle,cm}=1/2mr^2$) attached to the perimeter of the reel, a distance R from the center as shown. The reel spins freely, the fishing line does not stretch but moves frictionlessly through the guides along the rod, which does not bend and is held fixed.

(a) Find the moment of inertia of the reel and handle about its rotational axis.

The MOI of the reel has two contributions. One is the large cylinder, and since the axis of rotation is through the center of mass, this contribution is simply $I_{cyl,cm}=1/2MR^2$. *The handle however rotates about an axis which does not go through its COM, so we use the parallel axis theorem to find the MOI about the relevant axis, which is parallel and a distance R away.*

$$I_{reel} = \frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2$$

(b) A fish bites, is hooked, and accelerates in such a way that the tension in the line is 1N. Find the acceleration of the fish.

There is a known force applied at a perpendicular distance R from the axis of rotation, creating a known torque, and resulting in an angular acceleration of the reel about its axis.

$$|\vec{\tau}| = |\vec{r} \times \vec{F}| = RT = I_{reel}\alpha = I_{reel}\frac{a}{R} \qquad a = \frac{R^2T}{I_{reel}}$$

(c) The "drag" acts like a brake, completely stopping the rotation of the reel unless a minimum torque τ_{min} is applied. If the drag was set to τ_{min}=0.33Nm, could the fish in part (b) accelerate at all? Explain your answer.

This "drag" thing acts a bit like static friction, but for the case of rotational motion.

The torque created by the fishing line is *RT*=(0.06 m)(1N)=0.06 Nm, which is less than the critical drag of 0.33Nm, so the fish cannot accelerate at all. For a net acceleration, the tension in the line would have to be greater than 0.33Nm/0.06m=5.5N