Midterm #1 Solutions

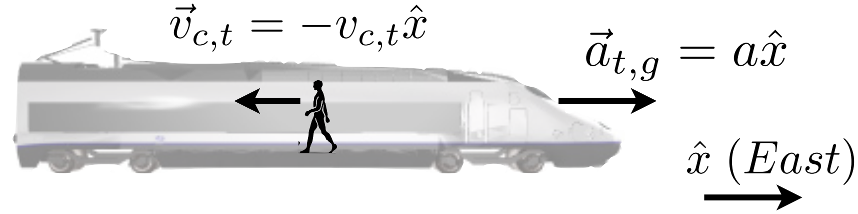
Physics 1501Q, Sections 21L-26L, Fall 2013

There are often many ways to arrive at an answer. For some conceptual problems, it happens through the intuition we have gained since birth about how the natural world works. For problems which use mathematics, there are often multiple approaches that works equally well. This solution set is only one possible account of reasoning through the test.

Inner monolog in italics

1. A train, starting from rest at position x=0 at time t=0, accelerates to the east with a constant acceleration $a_{t,g}$. A conductor walks from the front of the train toward the back at a constant speed $v_{c,t}$ with respect to the train.

A drawing could be helpful. For this, g=ground, t=train, and c=conductor. Important concepts are 1D kinematics (Unit 1) and relative motion (Unit 3).



(a) Write an equation of motion *x(t)* for the train using the following conventions: *x* is measured in meters and increases to the east and time is measured in seconds. Include units.

Hmmm, time dependent position along a track... This is a 1D constant acceleration kinematics problem (see Unit 1)!

I am going to need these:

$$a = const.$$
$$v(t) = at + v_0$$
$$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$$

OK, so since the train starts from rest at t=0,

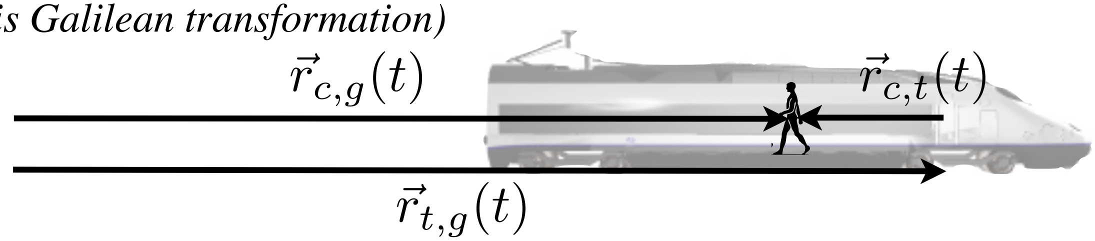
$$x(t=0) = 0$$
$$v(t=0) = 0$$

Which works if $x_0 = 0$ *and* $v_0 = 0$, *leaving*

$$x(t) = \frac{1}{2}at^2 = x_{t,g}(t)$$

(b) Using the same conventions, write an equation of motion for the passenger as measured from the ground. Include units.

OK, something or someone is moving in or on something that is also moving... This is a relative motion problem. Relative velocities, positions, and accelerations are related by addition (FYI-the fancy term is Galilean transformation)



$$\vec{v}_{c,t}(t) = -v_{c,t}\hat{x}$$
$$\vec{r}_{c,t}(t) = -v_{c,t}t\hat{x}$$

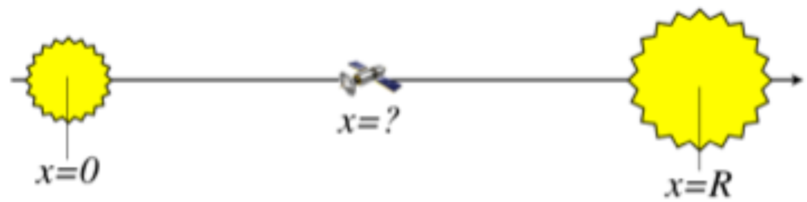
$$\vec{r}_{c,g}(t) = \vec{r}_{t,g}(t) + \vec{r}_{c,t}(t)$$
$$x_{c,g}(t) = x_{t,g}(t) + x_{c,t}(t)$$
$$x_{c,g}(t) = \frac{1}{2}at^2 - v_{c,t}t$$

As stated, numerical substitution and complete and correct units are required for full credit.

(c) While continuing to walk at constant speed with respect to the train, the conductor gently lifts his pocket watch so that it hangs motionlessly. Does the hanging watch lean toward the front of the train, toward the back of the train, or does it hang vertically?

If we were in a car and accelerated toward the east, the fuzzy dice would move appear to be pulled toward the back of the car (west) in our non-inertial reference frame (Unit 3). Sometimes this is called an "inertial force". Replace dice in car with watch on train, same deal. Because the conductor is moving at constant speed, there is no additional "inertial force" from his/her motion, so the answer doesn't change. The conductor would also feel an apparent pull toward the back of the train while walking, which would likely lead to a longer stride, and even stumbling if the acceleration were great enough. The answer is: The watch hangs toward the back of the train.

2. A binary star system consists of a star α with mass M_α and a star β with a mass $M_\beta=4M_\alpha$. If α is at location $x=0$, and β is at position $x=R$, at what location is the force of gravity on a satellite zero?

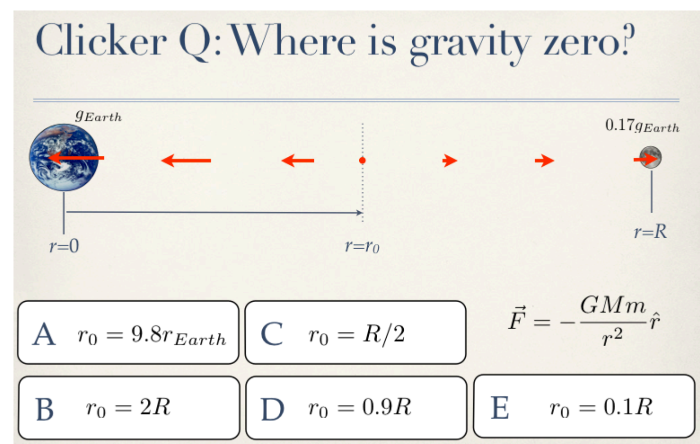


Gravity. OK, what is the formula again? It gets stronger when the masses increase, there is a fundamental constant of the universe G out front, and has r in the denominator. What is that clue up on the projector?

$$G = 6.67384(80) \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}} = 6.67384(80) \times 10^{-11}\ \mathrm{N\,(m/kg)^2}$$

Right! To make the units work, it has to be: $\vec{F}_G = -\dfrac{GMm}{r^2}\hat{r}$

A clicker question regarding where gravitational pulls from the two bodies cancelled was closer to the (lighter) moon. Universal gravitation was first covered in Unit 5.



Now, because gravity is attractive, the satellite of mass m_s is being pulled to the left by mass M_α a distance x away and pulled to the right by $4M_\alpha$ a distance R-x away in equal amounts.

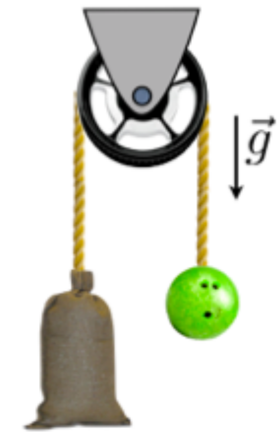
$$0 = -\frac{GM_\alpha m_s}{x^2} + \frac{G4M_\alpha m_s}{(R-x)^2}$$

$$x^2 = \frac{(R-x)^2}{4}$$

$$3x = R$$

$$x = \frac{R}{3}$$

3. A massless pulley hangs from the ceiling, and supports a massless rope. On one end of the rope, a mass M is attached and at the other end a mass $3M$ is attached. The two masses hang initially at the same elevation and are let go.
(a) What is the magnitude of the sum of all forces on the pulley?



FYI-this contraption is called an Atwood's Machine

Net force = mass times acceleration. The pulley never accelerates, though, so the net force must be ZERO. *The tensions in the left and right string segments which both pull downward must be accommodated by an upward force provided by the ceiling above.*

(b) What is the acceleration (direction and magnitude) of the lighter mass?

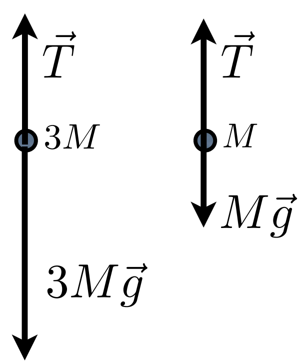
Asking for acceleration from near-earth gravity. Going to need a couple of free-body diagrams. Also a constraint that when one moves up, the other moves down by the same amount.

$$a_1 = -a_3 \quad \textit{(i)}$$

Analyzing the FBDs:

$$Ma_1 = T - Mg \quad \textit{(ii)}$$

$$3Ma_3 = T - 3Mg \quad \textit{(iii)}$$



Putting (i) in (iii),

$$3M(-a_1) = T - 3Mg \quad \textit{(iv)}$$

We need to get rid of T and find a_1.
Subtract independent equations (ii) from (iv):

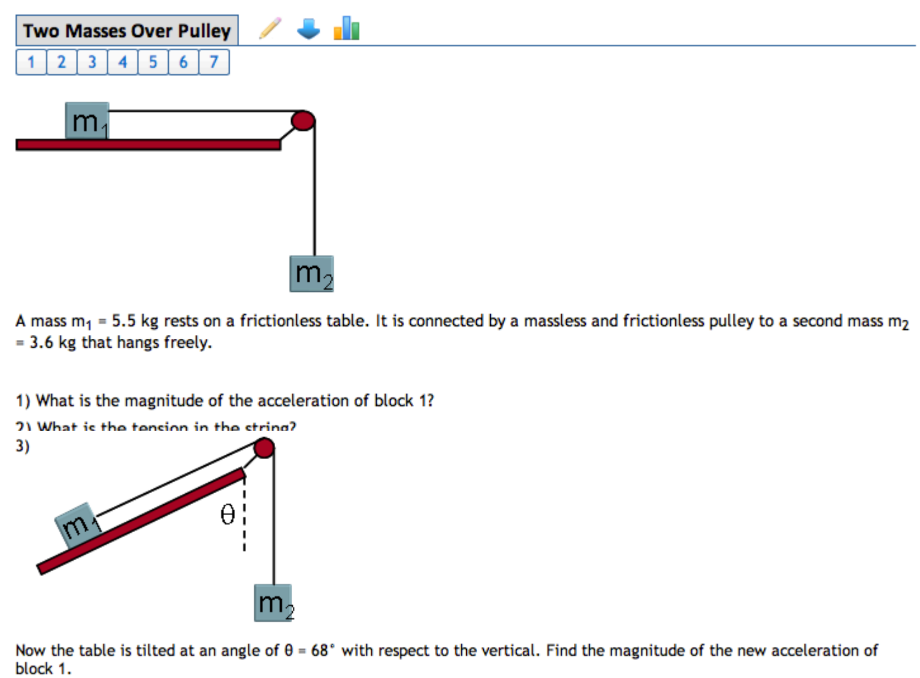
$$Ma_1 - 3M(-a_1) = T - Mg - T + 3Mg$$

$$4a_1 = 2g$$

$$a_1 = \frac{g}{2}$$

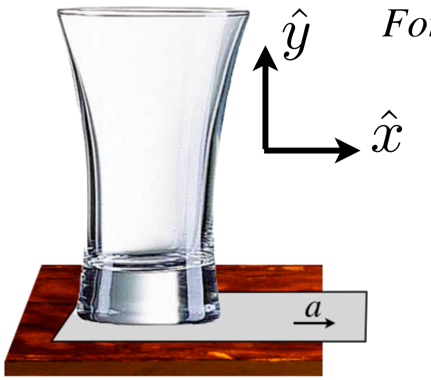
and for the direction,
of course the lighter mass goes upward.

There are two moving masses attached by a string, a much simpler version of Two Masses Over a Pulley in Unit 5.

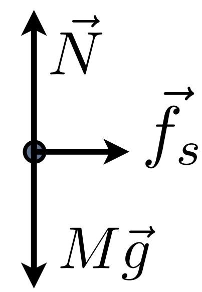


4. As a party trick, you pull a napkin with a uniform acceleration a from under a 0.5 kg glass. The coefficient of static friction between the glass and the napkin is μ_s=0.2.

(a) What is the minimum value a_c such that the glass slides under the napkin?



Forces again. Need a FBD.



y direction: $$Ma_y = 0 = N - Mg$$
$$N = Mg$$

x direction, no slipping:
$$Ma_x = f_s < \mu_s N$$

A slip occurs when
$$Ma_x = Ma = \mu_s N = \mu_s Mg$$
$$a = \mu_s g$$

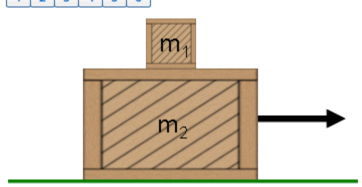
(b) How does this answer change if the glass is filled with 0.3 kg of marbles?

Friction increases with the mass, so M_{glass} is replaced by (M_{glass} + $M_{marbles}$) in the normal force, and so also the static frictional force. However, since now F=(M_{glass} + $M_{marbles}$)a, these will cancel in any determination of the critical acceleration for slipping. It can also be seen directly in the answer to (a) above. The answer is IT DOESN'T.

Problem 4 was taken from Unit 6 homework, Accelerating Blocks and Accelerating Truck.

Accelerating Blocks

1 2 3 4 5 6



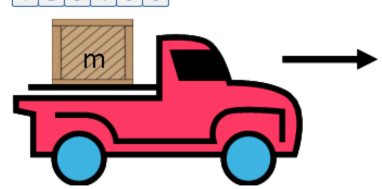
Two wooden crates rest on top of one another. The smaller top crate has a mass of m_1 = 17 kg and the larger bottom crate has a mass of m_2 = 93 kg. There is NO friction between the crate and the floor, but the coefficient of static friction between the two crates is μ_s = 0.82 and the coefficient of kinetic friction between the two crates is μ_k = 0.63. A massless rope is attached to the lower crate to pull it horizontally to the right (which should be considered the positive direction for this problem).

1) The rope is pulled with a tension T = 319 N (which is small enough that the top crate will not slide). What is the acceleration of the small crate?

2) In the previous situation, what is the frictional force the lower crate exerts on the upper crate?

Accelerating Truck

1 2 3 4 5 6

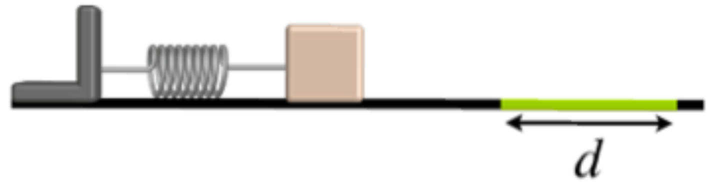


A box rests on top of a flat bed truck. The box has a mass of m = 15 kg. The coefficient of static friction between the box and truck is μ_s = 0.8 and the coefficient of kinetic friction between the box and truck is μ_k = 0.65.

1) The truck accelerates from rest to v_f = 16 m/s in t = 16 s (which is slow enough that the box will not slide). What is the acceleration of the box?

2) In the previous situation, what is the frictional force the truck exerts on the box?

5. A 1 kg box is launched across a floor, which is frictionless except for a rough patch d=1 m long where the coefficient of friction is μ_k=0.3. The launcher is a spring with constant k=10,000 N/m. How far would you have to compress the spring to successfully launch the box all the way through the rough patch?



Work done on block by spring=Change in block's kinetic energy due to spring decompression

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 - \frac{1}{2}m0^2$$

Work done by friction=Change in block's kinetic energy due to friction

$$-f_k d = -\mu_k mgd = \frac{1}{2}m0^2 - \frac{1}{2}mv^2$$

1) How much work is done by the spring as it accelerates the mass?

$$\mu_k mgd = \frac{1}{2}kx^2$$

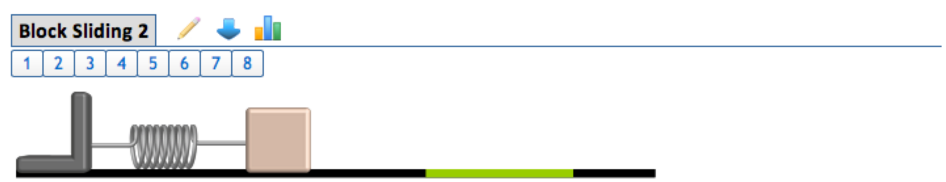
$$x = \sqrt{\frac{2\mu_k mgd}{k}}$$

This problem was taken directly from the homework Unit 7, Block Sliding and Block Sliding 2

A block with mass m = 13 kg rests on a frictionless table and is accelerated by a spring with spring constant k = 5119 N/m after being compressed a distance x_1 = 0.448 m from the spring's unstretched length. The floor is frictionless except for a rough patch a distance d = 2.3 m long. For this rough path, the coefficient of friction is μ_k = 0.49.

1) How much work is done by the spring as it accelerates the block?

2) What is the speed of the block right after it leaves the spring?

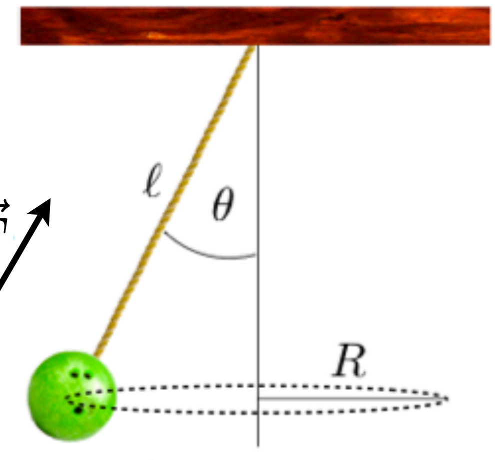


A mass m = 17 kg rests on a frictionless table and accelerated by a spring with spring constant k = 4597 N/m. The floor is frictionless except for a rough patch. For this rough path, the coefficient of friction is μ_k = 0.41. The mass leaves the spring at a speed v = 3.5 m/s.

1) How much work is done by the spring as it accelerates the mass?

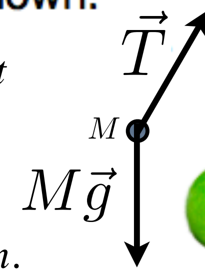
2) How far was the spring stretched from its unstreched length?

6. A ball of mass M = 5 kg is suspended from a long rope whose other end is attached to the ceiling. The ball travels in a horizontal circle of radius R = 1 m at a constant speed of v = 2 m/s.



(a) Draw the free-body diagram for the ball in a vertical plane as shown.

Careful: "centrifugal force", like "inertial force" are not real forces, but just mass time acceleration in a non-inertial reference frame. Here is the FBD:



(b) What is the magnitude of the net force on the ball?

The magnitude of the net force is the magntiude of the mass times acceleration. The accceleration of the ball is centripetal, pointing in towards the center of the circle.

$$F_{NET} = M\frac{v^2}{R}$$

(c) If the mass were doubled while keeping the speed constant, how would the angle θ change?

$$\tan\theta = \frac{|T_x|}{|T_y|}$$

If we double the mass, we double the vertical component of the tension because it has to cancel the downward gravitational force. If we double the mass, we also double the centripetal acceleration, and hence the horizontal component of the tension. The ratio of components of tension is therefore independent of the mass, and therefore so is tan theta, and therefore theta. You can also go through with a calculation, like below in (d), or have recognized the pattern that in other pendulum-in-uniform-gravity problems, masses drop out when motion is concerned. The answer is IT WOULDN'T.

(d) What is the length ℓ of the rope?

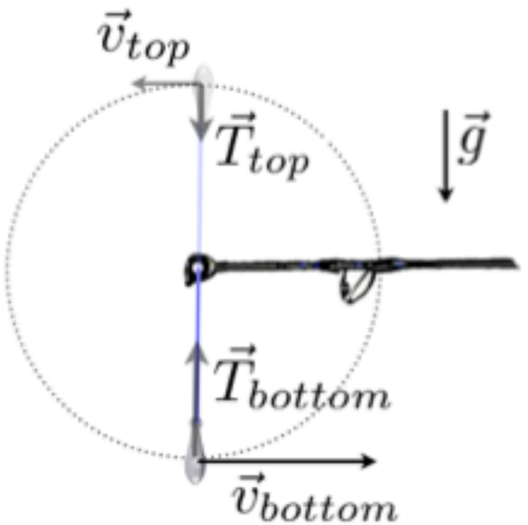
We know R, so if we knew theta, we could find l through trigonometry: $\ell = \dfrac{R}{\sin\theta}$
Theta is determined by a balance of forces and the cetripetal acceleration.
From the FBD,

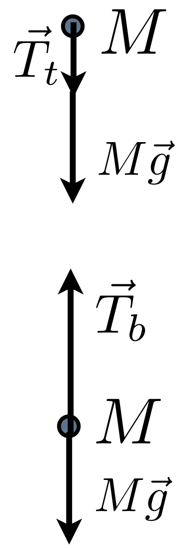
x direction: $M\dfrac{v^2}{R} = T\sin\theta$

y direction: $0 = T\cos\theta - Mg$

$$\tan\theta = \frac{v^2}{gR}$$

7. A 10 g weight is tied to the end of 0.25 m of string extending from a fishing rod tip. After a kick, it swings in a circular path in the vertical plane under the influence of gravity, which is pointed downward. The tension in the string at the bottom of the circular trajectory is twice that at the top ($|T_{bottom}|=2|T_{top}|$). What is the speed v_{bottom} of the weight at the bottom of the orbit? (*Hint: relate tensions to speeds using forces and then use the work-energy theorem to relate speeds*)





Forces:
At the top, the centrepital acceleration is down:

$$-M\frac{v_t^2}{R} = -T_t - Mg \qquad (i)$$

At the bottom, the centrepital acceleration is up:

$$M\frac{v_b^2}{R} = T_b - Mg \qquad (ii)$$

$$M\frac{v_b^2}{R} = 2T_t - Mg \qquad (iii)$$

The problem asks for speeds, so we have to eliminate tensions. Adding 2×(i) and (iii),

$$M\frac{(v_b^2 - 2v_t^2)}{R} = -3Mg$$

$$2v_t^2 - v_b^2 = 3Rg$$

Work-energy theorem:
Change in kinetic energy=-Change in potential energy

$$\frac{1}{2}M(v_t^2 - v_b^2) = -Mg(2R)$$

$$v_t^2 = v_b^2 - 4gR$$

$$v_b^2 = 11gR$$