

Letters to the Editor

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Temporal analysis of short laser pulses using degenerate four-wave mixing

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The temporal characteristics of picosecond and subpicosecond laser pulses have been investigated with a number of different techniques.^{1,2} Streak cameras provide the temporal profile of an optical pulse directly but are limited to a time resolution of ~ 1 psec. The two-photon fluorescence method³ using two counterpropagating replica pulses suffers from a one-pulse-induced background fluorescence that limits the precision of the temporal measurement. By using second-harmonic generation (SHG), a background-free measurement of the pulse duration can be made. In the original version,⁴ a variable time delay is introduced between the two replicas of the original pulse and for a strictly reproducible pulse train yields the time autocorrelation function of the original pulse. This method is limited to measuring the average characteristics of the pulse train, and, because of phase-matching considerations, the geometry must be changed for different optical wavelengths. A more recent version of the SHG method that is suitable for single-pulse measurements uses a time-to-space mapping of the interaction region of two noncollinear replica pulses.^{5,6} Again this technique is limited because of phase-matching problems.

In this Letter we introduce a new method for the measurement of the temporal profile of a single optical pulse that is based on degenerate four-wave mixing (DFWM).⁷ Among the numerous applications of DFWM,⁸ that most closely related to our present proposal is the time-domain correlator described by O'Meara and Yariv.⁹ Our technique produces a background-free measurement and has none of the phasematching difficulties associated with the SHG methods. Simple changes in the length of the nonlinear medium will allow measurements of pulse widths to be made in the subnanosecond to subpicosecond time interval.

We assume that two replica pulses of the original optical pulse are constructed with a simple beam splitter and mirror arrangement. The two replicas, with complex envelope functions $A_1(\mathbf{r},t)$ and $A_2(\mathbf{r},t)$, counterpropagate in a nonlinear optical medium as shown in Fig. 1. A quasi-cw probe field $A_4(\mathbf{r},t)$ illuminates the overlap volume of A_1 and A_2 . The probe field is of frequency ω , equal to the carrier frequency of the pulse fields. The phase conjugate wave $A_3(\mathbf{r},t)$ originating in the overlap region will be shown to have a transverse spatial amplitude distribution which is characteristic of the temporal profile of the pulses A_1 and A_2 . The four coupled wave equations for A_1 , A_2 , A_3 , and A_4 decouple and simplify considerably by assuming nondepletion of A_1 , A_2 , and A_4 in the slowly varying envelope approximation. The single equation for the phase conjugate field A_3 is

$$\left(\frac{\partial}{\partial z} + \frac{k}{\omega}\frac{\partial}{\partial t}\right)A_3(\mathbf{r},t) = \frac{i2\pi\omega\chi}{cn}A_1A_2A_4^*,$$
 (1)

where χ is the instantaneous third-order nonlinear susceptibility. We assume arbitrary input amplitude functions of the form

$$A_{1}(\mathbf{r},t) = f(\omega t - kx)g(y,z),$$

$$A_{2}(\mathbf{r},t) = f(\omega t + kx)g(y,z),$$

$$A_{4}(\mathbf{r},t) = h(x,y).$$
(2)

Using these functions in Eq. (1) and assuming that the overlap region is completely contained in the nonlinear medium, we obtain a formal solution to Eq. (1) of

$$A_{3}(\mathbf{r},t) = \frac{i2\pi\omega\chi}{cn}h(x,y)\int_{-\infty}^{z}f[\omega t - k(z + x - z')]$$
$$\times f[\omega t - k(z - x - z')]g^{2}(y,z')dz'.$$
(3)

We will use the asymptotic limit of this solution, valid far from the interaction region where the upper limit of the integral goes to plus infinity. The energy distribution on the detector is then

$$W_{3}(x,y) = \frac{1}{8\pi} \int_{-\infty}^{+\infty} A_{3}(\mathbf{r},t) A_{3}^{*}(\mathbf{r},t) dt.$$
(4)



Fig. 1. Schematic diagram for pulse duration measurements by degenerate four-wave mixing. The spatial distribution of the conjugate wave A_3 contains information about the temporal structure of replicas A_1 and A_2 of the pulse to be measured. The energy contained in A_3 is deposited on the detector (photoplate, diode array, etc.) after reflection from the semitransparent beam splitter. To observe the near-field distribution of the conjugate beam, a one-to-one imaging objective lens should be positioned between the beam splitter and detector plane.

We consider two cases of practical interest for which Eq. (4) may be evaluated. In the first case, the transverse dimensions of the replica pulses are assumed to be small compared with their longitudinal dimension. In addition, we assume that the probe field is spatially constant throughout the overlap region. Then Eq. (4) becomes

$$W_3(x) \simeq \int_{-\infty}^{+\infty} I(\omega t - kx)I(\omega t + kx)dt, \qquad (5)$$

where I is the longitudinal intensity profile of the pulse. We note that W_3 is a symmetrized spatial image of this pulse intensity profile about x = 0, from which the pulse duration can be determined. In this limit of small transverse dimensions, a modified replica of the original pulse can be used as the probe field, which is especially convenient for measurements with tunable picosecond lasers.

For the second case, we consider Gaussian profiles for both the temporal and spatial envelopes of the replica pulses,

$$f = a \exp[-\alpha(\omega t \pm kx)^2],$$

$$g = \exp[-\beta k^2(y^2 + z^2)],$$
(6)

where α and β are related to the pulse width (FWHM) *T* and the beam waist *w* by $T = (2 \ln 2/\alpha \omega^2)^{1/2}$ and $w = (\beta k^2)^{-1/2}$. With these expressions in Eq. (3) and keeping the probe field spatially constant, the energy distribution of the conjugate wave on the detector is

$$W_3(x,y) \simeq \exp[-4k^2(\alpha x^2 + \beta y^2)].$$
 (7)

The spatial dimension (FWHM) H of this elliptical Gaussian measured along the x direction gives the pulse duration T simply as

$$T = \sqrt{2}H/u, \tag{8}$$

where for generality u is the group velocity of the pulses.

The time resolution of this DFWM method is limited by considerations similar to those found for the SHG time-to-space mapping method. $^{5,10-12}$ The temporal spreading of the pulse wave packet due to the dispersive nature of the medium clearly limits the precision of the measurement. This effect can be minimized by keeping the length of the medium as small as possible. The divergence and alignment of beams 1 and 2 do not have a first-order effect on the mapping, but the alignment should be optimized to ensure adequate strength of the conjugate field. Diffraction effects due to the finite size of the interaction region will broaden the image on the detector although beam walk-off problems are not present. For ease of measurement, the spot size on the detector can be increased by using a converging probe beam. The lens is best placed before the beam splitter of Fig. 1. The spectral content of the probe field should be sufficiently large to cover the components of the pulse to enhance the strength of the conjugate field. Given the broad frequency distribution of the fields and especially in the case of frequency chirped or non transform-limited pump pulses, the observation of the phase conjugate beam should be limited to near field to avoid substructures in the image related to the finite coherence time of the pump pulse.¹² To achieve a background-free measurement, backreflection of the probe beam onto the detector should be avoided by slightly tilting the surfaces of the medium. With these limitations in mind, we anticipate an experimentally achievable time resolution of better than 0.1 psec.

In summary, we have presented a method for investigating the temporal structure of short optical pulses using a timeto-space mapping technique employing DFWM. The method is suitable for single-pulse or accumulated multiple-pulse measurements and is background free. Since the technique has no phase-matching difficulties, the geometry is invariant to changes in the pulse wavelength. The temporal resolution is estimated to be <0.1 psec.

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Anisotropic 2-D scattering. 3: The effects of incident laser wavelength

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This Letter presents the continuation of the experimental works reported in Refs. 1-3. In Ref. 1 the effects of a finite medium on multiple 2-D scattering from latex paint particles were presented. Multiple scattering from a semi-infinite medium of spherical polystyrene particles of fixed diameter was reported in Ref. 2. An important finding of Ref. 2 was that the intensity distribution curves for different particle sizes collapse to an effective isotropic solution by using (1 – g) as a correlation parameter with the radial optical thickness. g is the asymmetry factor.⁴ In Refs. 1-3, the incident light was from a He–Ne laser $(0.6328 \,\mu\text{m})$, while in Refs. 1 and 2 the carrier of the scattering centers was double distilled water. The effect of the carrier (i.e., its index of refraction) on the scattering of red light was reported in Ref. 3. That is, in addition to suspending the scattering centers in water, high purity ethylene glycol was used. The results of that investigation³ were consistent with both Refs. 1 and 2, and the two different particle carriers produced essentially the same results. This would indicate that the carrier index of refraction appears to be a relatively insensitive parameter of the backscattered intensity.

As a next step, the effects of the irradiation wavelength on the multiple scattering in a medium of finite thickness were considered. Consequently, a He–Cd laser beam $(0.4414 \,\mu\text{m})$ was required to be incident normal to the scattering medium. In all other respects, particularly in terms of particle size $(0.261 \,\mu\text{m})$, particle carriers (distilled water and high purity ethylene glycol), and magnitude of particle concentration (optical depth requirements), this experiment was identical to that presented in Ref. 3.

Therefore, the purposes of this investigation were for incident blue light (1) to demonstrate again that the anisotropic intensity distribution curves will collapse to an effective isotropic solution for a given value of τ_0^* by using (1 - g) as a correlation parameter and (2) to detect the effect of the refractive index of the liquids on the data received. Thus by comparing these results with those of Ref. 3 it will be possible to determine if the change in the irradiation wavelength adversely affects the data received as compared to theory.

In this experiment, it was assumed that, except for the very slight absorption character of the water or ethylene glycol (liq), the particle–water or the particle–ethylene glycol solutions possess only scattering properties (s). Thus the optical thickness for this investigation was defined as

$$\tau = \tau_s + \tau_{\rm liq},\tag{1}$$

where

$$\tau_s = \eta C_s L. \tag{2}$$

Note that η (the volume of scattering particles per total volume of liquid plus scattering particles) and L (the radius or depth of the medium) are measurable. C_s (the effective scattering cross section per particle volume) must be calculated.

The laboratory apparatus used to acquire the experimental data is the same as that discussed in Refs. 1–3. Thus the experimental setup and procedure followed exactly parallels that described in Ref. 3 and involved acquisition of data sets, where the sets were distinguished by the volume of scattering particles, the liquid carrier used, and its depth.

The ethylene glycol index of refraction (1.472) was measured⁵; the indices of refraction of water (1.3313) and the polystyrene spheres (1.6201) were calculated.^{6,7} The asymmetry factor g and C_s were determined from Mie theory (for water g = 0.73 and $C_s = 27392$ cm⁻¹; for ethylene glycol g =



Fig. 1. Experimental (symbols) and theoretical backscattered nondimensional intensities vs the radial optical thickness for double distilled water medium with a black bottom and 0.4414- μ m incident light (\bullet , 5 ml; \blacktriangle , 10 ml of spherical particles solution).



Fig. 2. Experimental (symbols) and theoretical backscattered nondimensional intensities vs radial optical thickness for an ethylene glycol medium with a black bottom and 0.4414-µm incident light (●, 20 ml; ▲, 40 ml of spherical particle solution).

 $\begin{bmatrix} \frac{R}{R_{b}} \end{bmatrix}^{2} \frac{I}{I_{b}}$

Fig. 3. Experimental (symbols) and theoretical backscattered nondimensional intensities vs radial optical thickness for an optical depth parameter of 2, a black bottom, and $0.4414 \cdot \mu m$ incident light (\bullet , 5 ml in water; \blacktriangle , 10 ml in water; \blacktriangledown , 20 ml in ethylene glycol; \blacksquare , 40 ml in ethylene glycol).



Fig. 4. Experimental (symbols) and theoretical backscattered nondimensional intensities vs radial optical thickness in an ethylene glycol medium for an optical depth parameter of 2 and a black bottom $(\bullet, 5 \text{ ml with } 0.4414 \, \mu\text{m}; \bullet, 10 \text{ ml with } 0.4414 \, \mu\text{m}; \bullet, 5 \text{ ml with } 0.6328 \, \mu\text{m}; \blacksquare, 10 \text{ ml with } 0.6328 \, \mu\text{m}.$

0.75 and $C_s = 7403 \text{ cm}^{-1}$). Thus the relative indices of refraction of latex in water and ethylene glycol are 1.217 and 1.100, respectively. The corresponding size parameters $(n\pi d/\lambda)$ are 2.473 and 2.734. The liquid depth Z was calculated from an imposed restriction that the asymmetric optical depth τ_0 be limited to values of $\frac{1}{2}$, 1, 2, 4, 5, and 10 where

$$\tau_0^* = (1 - g)\tau_0 = (1 - g)\eta C_s Z. \tag{3}$$

Equation (3) was used under the assumption that $\tau_0 > \tau_{\text{liq}}$. (The resulting error was <5%.)

A detector probe was moved radially, and detector signal levels were recorded. These output voltages are proportional to the amount of backscattered power transmitted to the PMT via the detector probe. The incident radiant power level was measured periodically and was found to be stable for the time required to take the data.

The incident radiation was assumed to be scattered anisotropically in the particles-liquid suspension dependent upon the effective optical thickness. Raw data pairs (voltage v and radial distance R) were recorded for various depths, liquid type, and dispensed particle volumes. A data reduction program from Ref. 8 transformed this raw data into a nondimensional scattered intensity $(R/R_b)^2(I/I_b)$ after processing input parameters of incident laser beam power level (PL), PMT dark level voltage, effective laser beam radius R_b , acceptance angle, and probe barrel radius. The program output is the following nondimensional scattered intensity as a function of radial optical thickness τ_R^* (with optical depth τ_0^* as parameter):

$$\left(\frac{R}{R_b}\right)^2 \frac{I}{I_b} = 538.86 v R_b^2 \ 10^{-6} / \text{PL}.$$
(4)

Figures 1 and 2 illustrate the program output. The heavy lines are theoretical results.⁹ These figures point out that the experimentally determined quantities do not exactly match the theoretical results (i.e., radial optical thickness are larger than the corresponding theoretical values and the experimental nondimensional intensities are low). Even so, the appropriate trends are exhibited by the experimental data in terms of slope, shape as a function of τ_R^* , and separation due to τ_0^* . Figure 3 is a typical comparison of $(R/R_b)^2(I/I_b)$ vs τ_R^* for both liquids for given optical depths τ_0^* . Notice that there is fair agreement for all the data presented.

Figure 4 is included to demonstrate that the incident radiation wavelength does not adversely affect the backscattered radiation. This particular figure is typical for ethylene glycol—for water there is a little more spread in the data. The $0.6328-\mu$ m data are from Ref. 3.

The data follow the same trends as predicted by the theory, although not exactly. The errors which cause these discrepancies appear to be independent of the medium and the irradiation wavelength. They are assumed to be due to the absorption of the particles and the scattering of the liquid.

In comparing the results found using water as the medium with those of ethylene glycol (Fig. 3), the results are very similar. Also Fig. 4 indicates that the data presentation is not a function of this incident radiation wavelength. Thus, for this wavelength (0.4414 μ m), as it was for the 0.6328- μ m incident wavelength of Ref. 3,

(1) the anisotropic nondimensional intensity distribution depends upon the asymmetry factor g, and, even though the range tested is small, (1 - g) is a correlation parameter in anisotropic backscattering from a medium of finite depth as it was with the anisotropic backscattering a semi-infinite medium; and

(2) the particle carriers produce very similar results.

Therefore, it may be concluded that changing of the inci-

dent radiation wavelength does not adversely effect the received data or the agreement between experimental and theoretical intensities.

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High-speed retardation modulation ellipsometer: erratum

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Equation (12) of this paper¹ was misprinted. It should be as follows:

$$\delta \Psi = \frac{\sum_{j=1}^{m} \sqrt{\frac{\sum_{i=1}^{l} (\Psi_{aj} - \Psi_{ij})^2}{l}}}{m},$$
 (12)

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Significance of the extinction/backscatter ratio and the boundary value term in the solution for the two-component lidar equation

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Recently, Klett¹ provided a stable inversion solution for the lidar equation which yields the quantitative extinction profile as a function of range. Fernald² also proposed a so-

lution for the two-component lidar equation (including aerosol and molecular terms) in a more general form and qualitatively stated that this solution is most sensitive to errors in the aerosol extinction/backscatter ratio and boundary value at the calibration level when applied to moderately turbid atmospheres. This Letter describes the characteristics of this solution and dependence of its accuracy upon extinction/ backscatter ratio and boundary value by means of numerical simulation.

Fernald's solution² is written as follows:

$$\alpha_{1}(z) = -\frac{S_{1}}{S_{2}}\alpha_{2}(z) + \frac{X(z_{c}) \exp\left[2\left(\frac{S_{1}}{S_{2}} - 1\right)\int_{z}^{z_{c}}\alpha_{2}(z)dz\right]}{\frac{X(z_{c})}{\alpha_{1}(z_{c}) + \frac{S_{1}}{S_{2}}\alpha_{2}(z_{c})} + 2\int_{z}^{z_{c}}X(z) \exp\left[2\left(\frac{S_{1}}{S_{2}} - 1\right)\int_{z}^{z_{c}}\alpha_{2}(z')dz'\right]dz},$$
(1)

where α and S are the volume extinction coefficient and extinction/backscatter ratio, respectively, and the suffixes 1 and 2 are quantities related to aerosol and molecule, respectively. X(z) is the range-squared lidar signal, and z_c is the range from the lidar at which a boundary condition is assigned. Molecular extinction coefficients are, in general, considered to be known from available meteorological data or appropriate standard model atmospheres.³

From Eq. (1) it is clear that Klett's solution is a special case of Fernald's where the aerosol extinction/backscatter ratio S_1 is formally set equal to molecular extinction/backscatter ratio S_2 , provided that the power k is equal to unity in Eq. (6) of Klett's original paper.¹ The aerosol extinction/backscatter ratio S_1 , however, depends on the size distribution and complex refractive index of the aerosols and varies from ~ 10 to 100.⁴ Generally, the value S_1 is unknown, and an appropriate estimation is necessary. Furthermore, a boundary value for α_1 at $z = z_c$ must be known by, for example, the slope method from lidar measurements.

In the following, some numerical calculations are carried out confining the situation to lidar measurements in the horizontal direction. Molecular extinction and backscattering coefficients⁵ are derived from the U.S. Standard Atmosphere³ with the laser wavelength of $0.532 \ \mu m$. The distribution of aerosol properties are modeled as follows:

$$\hat{\alpha}_1(z) = T \cdot \alpha_2 \cdot \left[1 + A \sin\left(\frac{2\pi z}{L} + \varphi\right) \right],$$
$$\hat{\beta}_1(z) = \hat{a}_1(z)/\hat{S}_1,$$

where $\hat{}$ indicates the model (true) quantity, $\hat{\beta}_1$ is the aerosol backscattering coefficient, T is an index for turbidity relative to the molecular extinction. L (=5 km), A (=1.0), and φ (=0.0) are the horizontal scale, the amplitude, and the phase of the aerosol distribution. The lidar signal is modeled with these quantities substituted into the usual lidar equation and is utilized for solving Eq. (1) under various S_1 and boundary value conditions. When solving Eq. (1), the backward integration formula given by Fernald² is used with a range step $\Delta z = 10$ m from the boundary at the range of 10 km.

Figure 1 shows an example of such calculations. It indicates that the amplitude of the solved profile becomes larger when S_1 increases and vice versa: Klett's solution gives a smaller amplitude than that given by the model. When the assumed boundary value differs from the true one, the solution converges to the profile for the true boundary value as the integration step goes backward from the boundary, and the con-



= 50 and T = 10; solution profiles with S_1 = 90 and 8.5 (= S_2). Boundary values are correctly given.

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vergence speed is faster as the turbidity and S_1 increase, although this is not shown here.

Reconstruction errors in the solution can be calculated using the following expression:

$$=\frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N} [\alpha_{1}(z_{i}) - \hat{\alpha}_{1}(z_{i})]^{2}}}{\frac{1}{N}\sum_{i=1}^{N} \hat{\alpha}_{1}(z_{i})}.$$

The value of ϵ is a function of N because the degree of convergence depends mainly on the assumed boundary value and the distance from the boundary. In the present calculation, the summation is done over the entire range (N = 1000).

Reconstruction errors were calculated for various combinations of four parameters: (1) turbidity T; (2) true extinction/backscatter ratio \hat{S}_1 ; (3) assumed extinction/backscatter ratio for Eq. (1) S_1 ; and (4) ratio of the assumed boundary value to the true one B_c . Some examples are shown in Fig. 2 for the cases of T = 5 (solid curve) and T = 20 (dashed curve). It can be seen from Fig. 2 that the solution is less sensitive to the boundary value ratio B_c in turbid air (T = 20)than in clean air (T = 5). Similarly, the solution has a weak dependence on the difference of S_1 from \hat{S}_1 in turbid air. This is why Klett's solution is better applied to turbid air.

The general tendency that the principal axes of the contours incline down on the right in this case implies that overesti-



Fig. 2. Reconstruction errors for the cases of T = 5 (solid curve) and T = 20 (dotted curve). From the top to the bottom, the contour maps represent the results corresponding to the conditions that S = 8.5 (Klett's solution), 30, 60, 90. The abscissa is the ratio of the assumed boundary value to the true one, and the ordinate is the true extinction/backscatter ratio in the model. Figures in the contour map are relative errors (in percent).

mates of the boundary value are preferred to underestimates when $S_1 < \hat{S}_1$ (as in Klett's solution)⁶ and vice versa: since the features of the contour map, however, are also dependent upon the aerosol distribution itself, general conclusions can be hardly derived.

When analyzing actual lidar signals using Eq. (1), it is necessary to introduce the proper S_1 and boundary value $\alpha_1(z_c)$ to decrease the solution error. Since it is impossible to know the true \hat{S}_1 value from the usual lidar measurements alone, it is necessary to estimate S_1 as close to the true \hat{S}_1 as possible, and there is no advantage to adopting Klett's solution with $S_1 = S_2$.

One effective method to estimate a plausible S_1 value which minimizes the probable error in the solution may be to select a S_1 which maximizes the following integral:

$$I(S_{1}) = \int \frac{1}{\epsilon(\hat{S}_{1}, S_{1})} P(\hat{S}_{1}) d\hat{S}_{1},$$

where $P(\hat{S}_1)$ is the probability that the true extinction/ backscatter ratio equals \hat{S}_1 . Thus it is very important to know the probability $P(\hat{S}_1)$ in relation to meteorological and atmospheric optical conditions.

As for the boundary value, it is essential to give precise values to the boundary term including $X(z_c)$ in Eq. (1) as well

as to $\alpha_1(z_c)$. The aerosol extinction coefficient for the boundary condition is usually estimated by the slope method, which is necessarily an averaged quantity over some range. Consequently, it is important to introduce an averaged value for $X(z_c)$ over the same range interval to reduce the boundary term error due to local inhomogeneities and noise. To this end, the following smoothing procedure would be effective:

$$\overline{X(z_{ic})} = \left[\frac{1}{M}\sum_{i=i_1}^{i_2} X(z_i) \exp(2 \cdot \overline{\alpha} \cdot Z_i)\right] \exp(-2 \cdot \overline{\alpha} \cdot z_{ic}),$$

where $M = (i_2 - i_1 + 1)$ and $i_c = (i_1 + i_2)/2$, and $\overline{\alpha} (= \overline{\alpha}_1 + \alpha_2)$ means the averaged extinction coefficient estimated by the slope method applied to the range from z_{i_1} to z_{i_2} .

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Millimeter wave gas/aerosol spectrophone and application to diesel smoke

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Recent developments in radiative sources at millimeter wavelengths, coupled with attendant advantages for propagation through aerosols, have resulted in a requirement for absorption coefficients at these wavelengths. The need is made more acute by the absence of complex indices and applicable calculational techniques. In response, prototype *in situ* millimeter wavelength spectrophone (or acoustical) systems have been developed by one of the authors (Bruce). The spectrophone design of this Letter is conceptually related to earlier gas-aerosol systems developed for use at the major IR transmission window wavelengths.^{1,2} This system was designed primarily with aerosols in mind but can be used to measure absorption by gases, aerosols, mixtures of these, or to separate the gaseous from the aerosol contribution.

Increasing the wavelength of the probe source toward millimeter wavelengths results in a rapidly decreasing scattered proportion for many aerosols of interest (to the fourth power of the particle dimension r) since the particulate size spectrum becomes Rayleigh $(2\pi r/\lambda \ll 1)$. Under the same condition, the magnitude of the absorption is often small rendering photoacoustical techniques, which have relatively high sensitivity, appropriate.

The millimeter wave system of this Letter is both acoustically and electrically tuned to obtain higher sensitivity per unit absorption and per unit microwave source power density. An impatt diode with ferrite isolator was used operating at 34-GHz wavelength. An output power of ~ 0.1 W was coupled first into a rectangular waveguide (WR-28) system where the wave was modulated and the forward power optimized. A flow-through cylindrical (circular) absorption cell was coupled to the rectangular portion of the waveguide through a transition. An electret microphone was coupled into the guide to detect the dominant acoustical mode. A ferrite modulator provided the time varying power and a reference signal for the phase sensitive detection system used. Forward and reverse waveguide power were controlled using an E/H tuner and crystal detectors mounted on a bidirectional (cross-guide) coupler.

Cylindrical absorption cells with various diameters have been used largely for the purpose of encouraging particular modes of propagation for the microwave energy. A desirable property for this energy is that its density (Poynting vector) be concentrated laterally in the center of the cell (waveguide cavity) rather than near the walls. This minimizes effects on the sensitivity which occur when particles collect on the cell wall. The cell used for the measurements of this paper had a 1.1-cm i.d. and 20-cm length. The aerosol was introduced at the end of the cell opposite the source of microwave power. This aerosol intake was designed to serve as a partial reflector for the formation of the electromagnetic and acoustical standing waves. Extraction of the aerosol at the opposite end of the absorption cell was accomplished through small holes in the circumference of the cylinder. A plenum over these holes and attached to the outer wall of the cylinder was connected to an aerosol filter and then to the metered flow system. Airflow velocities of \sim 50–70 cm/sec were employed for the aerosol absorption measurements. The intake portion of the cell was adjustable in position to permit tuning for resonance of the standing electromagnetic wave.

Longitudinal acoustical resonance was obtained by adjustment of the modulation frequency for the maximum acoustical signal with an absorbing gas in the cell. To prevent aerosols from circulating into the source region of the waveguide, a polymer window cut from commercial food wrap was inserted between waveguide flanges near the output end of the rectangular waveguide. A small counterflow of clean air was introduced between the window and absorption cell to prevent accumulation of aerosol particles in this region and particularly on the window.

The spectrophone system was calibrated using gaseous absorption. The extinction coefficients (which equal the absorption coefficients in the absence of appreciable scattering) of Freon 22 were measured in a one-pass transmission chamber³ which was carefully developed to be free of multipath effects which caused large errors in earlier results. The ratio of the resulting extinction coefficient to the acoustical cell detector signal voltage divided by a representative sample of the E-M wave power density in the guide determines the calibration factor. Calibrations performed at various levels of absorption confirmed linearity of the absorption with concentration in the range employed.

Signal to noise for a signal processing bandwidth of $\sim 10^{-1}$ Hz was found to be $< 0.05 \text{ km}^{-1}$ for a measurement (absorption signal minus base line signal), although, in the aerosol mode of operation, the airflow noise raises this to as much as $0.2-0.3 \text{ km}^{-1}$. While this is adequate for the purposes of this paper, a source of microwave power just received having 2.5 times the power used here is expected to increase the signal to noise by that proportion. The authors have observed that, without the flow-through requirement, the sensitivity can be considerably increased from the above values (i.e., by increasing cavity acoustical isolation, careful ratio-removal of source noise, and use of a higher cavity reflectivity or voltage standing wave ratio).

The smoke aerosol was generated externally, injected into a small aerosol chamber (volume \sim 70 liters), and stirred lightly throughout the measurements. The diesel fuel was burned on a short cylindrical wick (diam = 1.2 cm) immersed in a small cup having the same diameter. Above the wick, an aluminum cone funneled the smoke through a copper tube, cooling the smoke, into the chamber.

The aerosol chamber (volume \sim 70 liters) was sampled by the flow-through spectrophone and Gelman filter particle collection systems. Measurements of the smoke were continuously obtained using the spectrophone and periodically using the flow-through filter sampling technique. Separate aerosol filters were used for two purposes though the filter material, and pore sizes were the same, i.e., Nuclepore, polycarbonate with 0.2-µm pore size. One set of filters was used for determination of aerosol mass loading and the other for SEM photos of the particles forming the distribution. To determine the mass loading, each filter was placed in a small hermetically sealed container and weighed. Then the filter was inserted into the sampling system and a timed sample obtained. Posttest weighing, divided by the sampling time and the calibrated flow rate, yielded the mass density of the smoke. The absorption coefficient divided by this number gives the absorption per unit mass density, i.e., the mass absorption coefficient.

The second set of filters was loaded very lightly using low flow rates and short sampling periods to avoid deceptive appearances of *in situ* coagulation. A sample photo is shown in Fig. 1.

These particles appear predominantly to be stringy (a high length-to-radius ratio). Although those on the filters are generally of the order of $2 \,\mu$ m long, a few are quite large, including those with sufficient extent to be visible to the eye.

It was determined that the particle chemical content is largely carbonaceous (77.5% by weight). The remainder consists of high molecular weight hydrocarbons, mainly alkanes of the general formula C_nH_{2n+2} with $12 \le n \le 34.4$

The mass absorption coefficient for diesel smoke obtained from the measurements described is $6.1 \times 10^{-3} \text{ m}^2 \text{ g}^{-1}$, $\pm 4 \times 10^{-4} \text{ m}^2 \text{ g}^{-1}$, where the latter number is based on the repeatability of the net result. The estimated probable error is higher at 12% emphasizing systematic errors.

Possible absorption contributions due to gaseous products of the combustion process were then investigated by use of a



Fig. 1. Scanning electron photomicrograph of smoke particles. Bar length indicates $10 \ \mu m$.

removable filter between the smoke chamber and spectrophone. This filter was installed so that the insertion itself did not affect the system parameters. When the filter was in the flow circuit the absorption was negligible relative to the level previously obtained for the smoke. Measurement of the total absorption as a function of time after the smoke generation with periodic mass loading measurements yielded another test of the proportion of gaseous-to-total absorption. The total absorption per unit mass density was constant in time rather than increasing, implying that the gases do not significantly contribute.

Although it is desirable to obtain a value for the mass absorption coefficient from theory for comparison with that of the spectrophone, the lack of values for the complex refractive index and the complexity of the aerosol with regard to shape and composition (including inhomogeneities) render this very difficult. A very rough approximation can be obtained by assuming that the complex index for the carbonaceous aerosol is constant from visible through millimeter wavelengths. Comparisons of theory and measurements for similar smokes exist at visible⁵ (0.55- μ m) and IR^{5,6} (10.6- μ m) wavelengths. If the absorption coefficient is assumed to be inversely proportional to the wavelength,⁵ the extrapolated result is of the order of 10⁻³ m² g⁻¹. No explanation for the difference is apparent.

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- 6. C. W. Bruce and N. M. Richardson, Appl. Opt. 22, 1051 (1983).

Frequency domain measurements of lag on charge injection devices

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General Electric Company, Electronics Laboratory, Syracuse, New York 13221. Received 22 August 1983. 0003-6935/84/010015-03\$02.00/0. © 1984 Optical Society of America.

An ideal optical detector illuminated by an impulse of light would yield an electrical signal at the first reading and zero residual signal in subsequent readings. In reality, most detectors such as charge injection devices (CID) do not respond instantaneously but rather exhibit lag.^{1,2} When a CID pixel is exposed to a unit impulse of light and its output signal held from sample to sample in a sequence of reads, the waveform shown in Fig. 1 results. Notice that the first reading is not unity (assuming 100% detector performance efficiency), and the signal decays exponentially with base α . This α , which satisfies $0 < \alpha < 1$, is the quantity which we use to describe lag. The lag is of great concern for the CID as an optical detector, because it not only degrades the impulse response but also reduces the charge storage capacity of the detector.

The principle of the CID is that the photogenerated charge. integrated and stored in the metal-insulator-semiconductor (MIS) capacitors, is read out by collapsing the potential well which causes the charge to recombine in (or be injected into) the substrate, thereby inducing a current in an external circuit.¹ Such drive line sensing readout is usually configured to operate with a high impedance voltage preamplifier. The physical mechanism of lag in the CID with voltage type readout is twofold: readout lag and device lag. The former is due to the injection pulse loading effect inherent in the voltage sensing scheme, whereas the latter arises from incomplete charge injection or recollection of the injected charge because of the traps or sticky charge in the potential well. Empirical results of an exponential response to a step-function excitation suggest that the trapped charge after each injection is linearly proportional to the charge population in the potential well before injection. Such lag mechanism is essentially a linear system and can be described by a constant α .

An accurate measurement of lag in the time domain requires synchronization between the optical pulse and the CID pixel reading time. Furthermore, the residual signal due to lag must be large compared with the noise, which is especially demanding in the case of a low lag situation. Lag measurement in the frequency domain on the contrary is far superior. The idea is to analyze the way lag modifies the frequency response of the output of a detector such as CID. In this Letter, a derivation of the frequency domain lag model will be presented.

The CID is an integrating type of detector, and its output is normally displayed by a sample-and-hold circuit. Assuming a sinusoidal input signal (optical), the frequency response due to integration and sampling can be written as³

$$A_s(f) = \lim_{N \to \infty} \sum_{n=0}^N \frac{1}{N} a(nT) \exp(-j2\pi f nT), \tag{1}$$

where T is the sampling period, which corresponds to the integration time of the device, and a(nT) is the detected signal at t = nT given by



Fig. 1. Unit impulse response of a CID detector with $lag \alpha = 20\%$.



NORMALIZED FREQUENCY

Fig. 2. Normalized frequency response of a CID detector as a function of lag from 0 to 60%.



NORMALIZED FREQUENCY

Fig. 3. Empirical frequency response of two InSb CID infrared detectors with lags of 10 and 30%, respectively, measured in the time domain.

$$a(nT) = (1 - \alpha) \left\{ \int_{(n-1)T}^{nT} \exp(j2\pi ft) dt + \alpha \int_{(n-2)T}^{(n-1)T} \exp(j2\pi ft) dt + \alpha^2 \int_{(n-2)T}^{(n-2)T} \exp(j2\pi ft) dt + \ldots \right\}$$
$$= (1 - \alpha) \exp(j2\pi fnT) [T \exp(-j\pi fT)] \times \frac{\operatorname{sinc}(fT)}{[1 - \alpha \exp(-j2\pi fT)]}, \qquad (2)$$

where $\operatorname{sinc}(x) = \sin(\pi x)/\pi x$. The first term in the braces of

Table I.	Table I. Two-Frequency Response Ratio vs Lag	
Las (%)	Ø.1/Ø.3	Ø.2/Ø.4
Ø	1.313Ø8	1.52786
2	1.343Ø7	1.562Ø6
4	1.37433	1.59628
6	1.4069	1.63046
8	1.4408	1.66455
1Ø	1.47605	1.69849
12	1.51269	1.73221
14 16 18	1.530/3 1.5902 1.63111	1.79877 1.83149
2Ø 22 24	1.67348	1.86375 1.8955
24 26 28	1.80934 1.85753	1.95725 1.98714
3Ø	1.90716	2.Ø163
32	1.95818	2.Ø447
34	2.01055	2.Ø7229
36	2.Ø6423	2.09903
38	2.11915	2.12489
40	2.17522	2.14983
42	2.23235	2.17384
44	2.29Ø43	2.19688
46	2.34932	2.21894
48	2.40887	2.24001
50	2.46882	2.24000
52	2.52926	2.27914
54	2.58969	2.29719
56	2.64998	2.31423
58	2.7Ø988	2.33027
60	2.76913	2.34532
62	2.82745	2.35939
64	2.88455	2.3725
66	2.94014	2.38466
68	2.99393	2.3959
7Ø	3.04562	2.40623
72 74 ·	3.09494 3.1416 3.10537	2.4157 2.42431
78 8Ø	3.226Ø1 3.26334	2.43209 2.43909 2.44532
82	3.2972	2.45081
84	3.32745	2.4556
86	3.35401	2.45971
38	3.37685	2.46318
90	3.39594	2.46604
94 96	3.41133 3.42309 3.43131	2.46831 2.47002 2.47122
98 Note: The freq	3.43613	2.47191

frequencies f_ct_I.

Eq. (2) is the signal integrated in the primary cycle, whereas the subsequent terms represent the delayed residual signal due to lag from previous cycles. The factor $(1 - \alpha)$ accounts for the lag for the corresponding cycle t = nT.

We can now bring the effect of zero-order hold into the expression by writing $A(f) = A_s(f)H_0(f)$, where $H_0(f)$ is the transfer function of the zero-order hold with holding time T given by³ $H_0(f) = T \operatorname{sinc}(fT) \exp(-j\pi fT)$, and the response becomes

$$A(f) = (1 - \alpha)T^2 \exp(-j2\pi fT) \frac{\operatorname{sinc}^2(fT)}{[1 - \alpha \exp(-j2\pi fT)]}$$

The normalized amplitude frequency response for the whole system can then be expressed as

$$|A_n(f)| = \frac{|A(f)|}{|A(0)|} = \frac{(1-\alpha)\sin^2(fT)}{\sqrt{(1-\alpha)^2 + 4\alpha}\sin^2(\pi fT)} \,. \tag{3}$$

With the implied normalization understood, the subscript n of $A_n(f)$ can be dropped. A plot of Eq. (3) with α as a parameter is shown in Fig. 2, where the horizontal axis is the normalized frequency $f_c t_I$, f_c being the chopping frequency and t_I the integration time ($t_I = T$). By measuring the optical response at two chopping frequencies, the lag α can be deduced from the ratio

$$R(\alpha, f_1, f_2) = \frac{|A(f_1, \alpha)|}{|A(f_2, \alpha)|}$$
(4)

using an iterative computer routine. The need for signal response normalization is thus eliminated. Table I lists the two-frequency response ratio of Eq. (4) as a function of lag for normalized frequency pairs of 0.1/0.3 and 0.2/0.4. Depending on the integration time, the chopping frequency can be properly chosen for good resolution in the lag measurement.

Since the optical signal is normally modulated by a chopper in the optical laboratory, a wave analyzer (e.g., HP302A, Δf = 6 Hz) can be used to select only the fundamental frequency component. The measurement thus involves a pure sinusoidal waveform making automated lag measurements not only possible but also easy to do. As a test of this frequency domain lag model, Fig. 3 plots the frequency response of two indium antimonide CID infrarared detectors whose lag was measured to be 10 and 30%, respectively, using the impulse reponse method (time domain measurement). The fit of data points into the theoretical curves is remarkable, and the agreement is excellent between frequency and time domain approaches.

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References

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Meetings Calendar

January

- 9–11 Experimental Mechanics of Fiber Reinforced Composite Materials seminar, San Diego SESA, 14 Fairfield Dr., Brookfield Ctr., Conn. 06805
- 9-20 Optical Science & Eng. course, Tucson P. Slater, P.O. Box 18667, Tucson, Ariz. 85731
- 16–20 Laser Beam Propagation & Interaction Effects course, Huntsville Eng. Tech., Inc., P.O. Box 8859, Waco, Tex. 76714
- 22–27 SPIE Technical Symp., Los Angeles SPIE, P.O. Box 10, Bellingham, Wash. 98227
- 23–25 Optical Fiber Communication, OSA/IEEE Top. Mtg., New Orleans OSA, Mtgs. Dept., 1816 Jefferson Pl., N.W., Wash., D.C. 20036
- 25–27 31st Ann. Western Spectroscopy Assoc. Conf., Pacific Grove D. Saperstein, IBM Instruments, 40 W. Brokaw Rd., San Jose, Calif. 95110

February

- 12-15 Color & Imaging Conf., Williamsburg F. Billmeyer, Jr., Chem. Dept., Rensselaer Poly. Inst., Troy, N.Y. 12181
- 13-17 Laser Fundamentals & Systems course, Dallas Eng. Tech., Inc., P.O. Box 8859, Waco, Tex. 76714
- 26–29 Application of Optical Instrumentation in Medicine XII: Medical Image Production Processing, Display, & Archiving & Exhibit, San Diego R. Schneider, SPIE, P.O. Box 10, Bellingham, Wash. 98227

March

3–5 Chemical Lasers course, Las Cruces. Eng. Tech., Inc., P.O. Box 8859, Waco, Tex. 76714

- 5–7 Laser Techniques in the Extreme Ultraviolet, OSA Top. Mtg., Boulder OSA, Mtgs. Dept., 1816 Jefferson Pl., N.W., Wash., D.C. 20036
- 5–9 1984 Pittsburgh Conf., Atlantic City L. Briggs, 437 Donald Rd., Pittsburgh, Pa., 15235
- 11-16 Microlithography & Exhibit Conf., Santa Clara A. Wagner, SPIE, P.O. Box 10, Bellingham, Wash. 98227
- 13–15 Chemical Lasers course, Las Cruces Eng. Tech., Inc., P.O. Box 8859, Waco, Tex. 76714
- 19–21 Phase Conjugation Methods course, San Diego Eng. Tech., Inc., P.O. Box 8859, Waco, Tex. 76714
- April

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- Optika '83 Symposium, Budapest Optical, Acoustical & Filmtechnical Society, Budapest, Anker koz l., H-1060-Hungary
- 2-5 3rd Inter. SPSE Symp. on Photofinishing Technology, Las Vegas R. Wood, SPSE, 7003 Kilworth Lane, Springfield, Va. 22151
- 8–10 Medical Imaging & Instrumentation '84 Symp., Las Vegas J. Mulvaney, Radiology Dept., U. of Michigan Hospital, Ann Arbor, Mich. 48109
- 8–13 187th ACS Natl. Mtg., St. Louis A. T. Winstead, 1155 16th St., N.W., Wash., D.C. 20036
- 11–14 The Frontiers of Optometry—Inter. Optometric Congr., London Congress Admin., The British College of Ophthalmic Opticians, 10 Knaresborough Place, London, SW5 OTG, England
- 14-20 Int. Optical Design Conf., Monterey OSA Mtgs. Dept., 1816 Jefferson Pl., N.W., Wash., D.C. 20036

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