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Quiz - 10

Given $x_P = 0.042m$, and $x_Q = 0.570m$, and $\sigma = 0.34\ \mu C/m$

a) $\vec{E}_0 = 1.92 \times 10^4 \frac{N}{C} \hat{x}$ is constant, find $V_Q - V_P$. Is V_P greater or lesser than V_Q?

Potential difference is $V_Q - V_P = -\int_{x_P}^{x_Q} \vec{E} \cdot d\vec{l}$

But $\vec{E}$ is entirely in the $\hat{x}$-axis and $|E_0|$ is constant so

$$V_Q - V_P = -\int_{x_P}^{x_Q} (|E_0|\hat{x}) \cdot (\hat{x}\, dx)$$

$$= -|E_0|\ x \Big|_{x_P}^{x_Q}$$

$$= -|E_0|\ (x_Q - x_P)$$

$$= -1.92 \times 10^4 \frac{N}{C}\ (0.57m - 0.042m)$$

$$\boxed{V_Q - V_P = -1.01 \times 10^4 \frac{N\text{-}m}{C}}$$

By this, the potentials satisfy

$$V_Q + 1.01 \times 10^4 \frac{N\text{-}m}{C} = V_P$$

$$\boxed{V_Q < V_P}$$

b) if $\vec{E} = |E_0| e^{-0.45x} \hat{x}$, calculate $V_Q - V_P$.

As before, potential difference is

$$V_Q - V_P = -\int_{x_P}^{x_Q} \vec{E} \cdot d\vec{l}$$

$$= -\int_{x_P}^{x_Q} (|E_0| e^{-0.45x} \hat{x}) \cdot (\hat{x}\ dx)$$

$$= -|E_0| \int_{x_P}^{x_Q} e^{-0.45x}\ dx$$

$$= \frac{-|E_0|}{-0.45} e^{-0.45x} \Big|_{x_P}^{x_Q}$$

$$= \frac{|E_0|}{0.45} \left(e^{-0.45x_Q} - e^{-0.45x_P} \right)$$

$$= \frac{1.92 \times 10^4 \frac{N}{C}}{0.45(\frac{1}{m})} \left(e^{-0.45\frac{1}{m} \cdot 0.570m} - e^{-0.45\frac{1}{m} \cdot 0.042m} \right)$$

$$\boxed{V_Q - V_P = -8,854 \frac{N\text{-}m}{C}}$$

Note that any quantity in an exponential must be unitless. Since x has length units, 0.45 must have units of inverse length ($\frac{1}{m}$) to end up with the correct units of volts.

Points	
a) $\Delta V = -\int_A^B \vec{E}\cdot d\vec{\ell}$	15
treat E as constant	5
Integrate correctly	5
correct $V_Q - V_P$	5
units	5
Attempt to answer $V_Q \overset{?}{<} V_P$	5
correct $V_Q < V_P$	5
b) Integrate correctly	5
correct $V_Q - V_P$	5
units	5
	60 + 40 free

Note, for part (a), it was equally correct to use charge distribution and Gauss' law to find $\vec{E}$. In the limit that the sheet is very large, take the surface to be a pill box of negligible width about the sheet. Then

$$\oint \vec{E}\cdot\hat{n}\, da = \frac{Q_{enc}}{\epsilon_0}, \qquad Q_{enc} = \oint \sigma\, da = \sigma A$$

$$|E|2A = \frac{\sigma A}{\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{x}$$

since $\epsilon_0 = 8.85\times10^{-12}\ C^2/N\text{-}m^2$,

$$\vec{E} = \frac{0.34\times10^{-6}\frac{C}{m^2}}{2\cdot 8.85\times10^{-12} C^2/N\text{-}m^2}\hat{x}$$

$$\vec{E} = 1.92\times10^4 \frac{N}{C}\hat{x}$$

which of course is just equal to the given E_0:

$$\vec{E} = E_0\hat{x}$$

for part (a) only.