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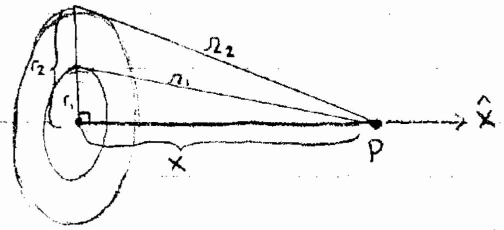
4/16/09 - Quiz 9

Two concentric rings of radii $r_1 = 0.5m$ and $r_2 = 0.75$ have uniform charge per unit length $\lambda_1 = -3\mu C/m$ and $\lambda_2 = 8\mu C/m$.



Calculate potential V due to both rings at point P located along the symmetry axis, $x = 1.25m$.

a) For a continuous distribution, potential is

$$V = \int k \frac{dq}{\mathscr{r}}.$$

From the geometry, Pythagorean Thm gives

$$\mathscr{r}_1 = \sqrt{r_1^2 + x^2} \quad \text{and} \quad \mathscr{r}_2 = \sqrt{r_2^2 + x^2}.$$

Since λ is a linear charge distribution,

$$\lambda = \frac{dq}{d\ell} \Rightarrow dq = \lambda \, d\ell.$$

But for an infinitesimal length along a circle, $d\ell$ is actually arc length so

$$d\ell = r\,d\phi$$

where ϕ is the angle about the ring. Therefore,

$$dq_1 = \lambda_1 r_1 \, d\phi \quad \text{and} \quad dq_2 = \lambda_2 r_2 \, d\phi.$$

Combining these,

$$V = \int k \frac{dq}{\mathscr{r}}$$

by superposition

$$= k\left[\int \frac{dq_1}{\mathscr{r}_1} + \int \frac{dq_2}{\mathscr{r}_2}\right]$$

$$= k\left[\int_0^{2\pi} \frac{\lambda_1 r_1 \, d\phi}{\sqrt{x^2 + r_1^2}} + \int_0^{2\pi} \frac{\lambda_2 r_2 \, d\phi}{\sqrt{x^2 + r_2^2}}\right]$$

$$= k\left[\frac{\lambda_1 r_1}{\sqrt{x^2 + r_1^2}} + \frac{\lambda_2 r_2}{\sqrt{x^2 + r_2^2}}\right] \int_0^{2\pi} d\phi$$

$$= 2\pi k\left[\frac{\lambda_1 r_1}{\sqrt{x^2 + r_1^2}} + \frac{\lambda_2 r_2}{\sqrt{x^2 + r_2^2}}\right]$$

Plugging in known values,

$$V = 2\pi \left(9.0 \times 10^9 \frac{N\text{-}m^2}{C^2}\right)\left[\frac{\left(-3\times10^{-6}\frac{C}{m}\right)(0.5m)}{\sqrt{(1.25m)^2 + (0.5m)^2}} + \frac{\left(8\times10^{-6}\frac{C}{m}\right)(0.75m)}{\sqrt{(1.25m)^2 + (0.75m)^2}}\right]$$

$$\boxed{V = 1.70 \times 10^5 \frac{N\text{-}m}{C}}$$

b) What does part (a) tell you about the direction of the electric field at point P?

In (a), we found that

$$V(x) = 2\pi k \left[\frac{\lambda_1 r_1}{\sqrt{x^2 + r_1^2}} + \frac{\lambda_2 r_2}{\sqrt{x^2 + r_2^2}} \right]$$

which indicates that as x increases, V(x) decreases. In other words, without explicit differentiation we can conclude that

$$\frac{dV(x)}{dx} < 0$$

or that $\frac{dV}{dx}$ is negative. Using the fact that electric field is

$$E_x = -\frac{dV}{dx}$$

in the x-direction and that $\frac{dV}{dx}$ is negative, it follows that the electric field must point in <u>the positive $\hat{x}$-direction, away from the plane.</u>

<u>POINTS</u>

a) $V = k \int \frac{dq}{r}$	15
identify r_1, r_2 correctly	10
find dq_1, dq_2	10
correct integration	10
plug in correctly	5
units	5
not a vector	10
<u>b) "Away from the plane", justified</u>	<u>10 pts</u>
	75 pts + 25 free