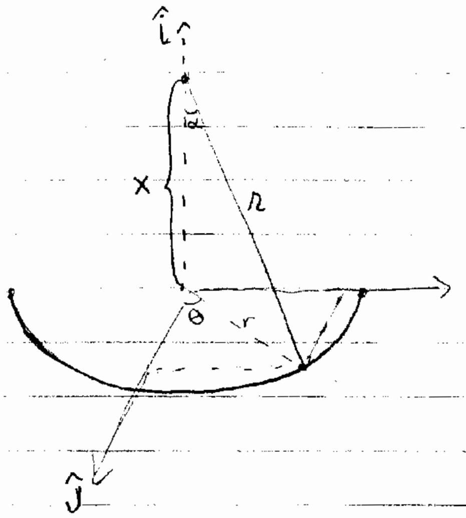
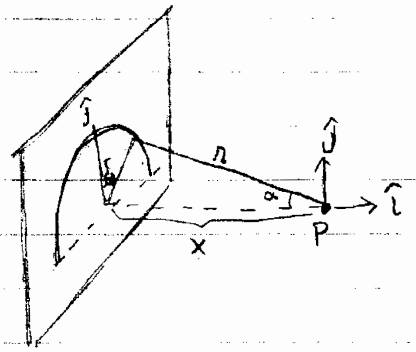
Kristen Basiaga
4|2|09 - Quiz 8



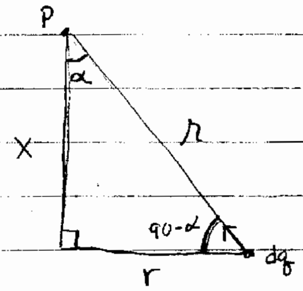


Given $\lambda = 0.025 \frac{C}{m}$, $r = 0.5$ m and P placed a distance x from the center of a ½ loop as shown, find the x-component of electric field.

In general, for a continuous charge distribution,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

where r is the separation between the point P and some infinitesimal charge, dq. From the geometry,



$$r = \sqrt{x^2 + r^2}$$

and the x-component of is proportional by

$$\sin(90-\alpha) = \cos(\alpha) = \frac{\text{Adj.}}{\text{Hyp.}} = \frac{x}{r}.$$

Thus some infinitesimal electric field in the x direction is

$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \frac{x}{r}$$

$$dE_x = \frac{x}{4\pi\epsilon_0} \frac{dq}{(x^2+r^2)^{3/2}}$$

We know that the charge per unit length determines dq:

$$\lambda = \frac{dq}{dl} \rightarrow dq = \lambda\, dl$$

where dl is the length along the wire. Since the length of a circular path is really arc length

$$l = r\theta \qquad \rightarrow \qquad dl = r\, d\theta$$

Replacing this in the previous expression,

$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{x}{(x^2+r^2)^{3/2}} \lambda r \, d\theta$$

Integrating both sides from 0 to π for a $\frac{1}{2}$-circle, this gives

$$E_x = \frac{\lambda x r}{4\pi\epsilon_0 (x^2+r^2)^{3/2}} \int_0^{\pi} d\theta$$

$$\boxed{\vec{E}_x = \frac{\lambda x r}{4\epsilon_0 (x^2+r^2)^{3/2}} \hat{x}} \qquad \left(\text{or} \quad \frac{\pi k \lambda x r}{(x^2+r^2)^{3/2}} \hat{x} \right)$$

Plugging in numerical values,

$$\vec{E}_x = \frac{\pi (9.0 \times 10^9 \text{ N-m}^2/C^2)(0.025 \frac{C}{m})(0.5m) \; x \; \hat{x}}{(x^2 + (0.5m)^2)^{3/2}}$$

$$\boxed{\vec{E}_x(x) \approx \frac{(3.534 \times 10^8 \text{ N-m}^2/C) \; x \; \hat{x}}{(x^2 + 0.25m^2)^{3/2}}}$$

Points Breakdown

general:	• Know equation for $\vec{E} = k \int \frac{dq}{\imath^2} \hat{\imath}$	15
geometry	• Identify $\imath = \sqrt{r^2+x^2}$	10
	• x-component	
	→ attempt to find it	5
	→ $E_x = \frac{x}{\imath} \lvert E \rvert$	5
charge distribution	• $\lambda = \frac{dq}{d\ell}$, find ~~dq~~	10
	• Arc length $d\ell = r\,d\theta$	5
plug + chug	• integrate $\int_0^{\pi} d\theta = \pi$	5
	• evaluate numerically	5
		60 + 40